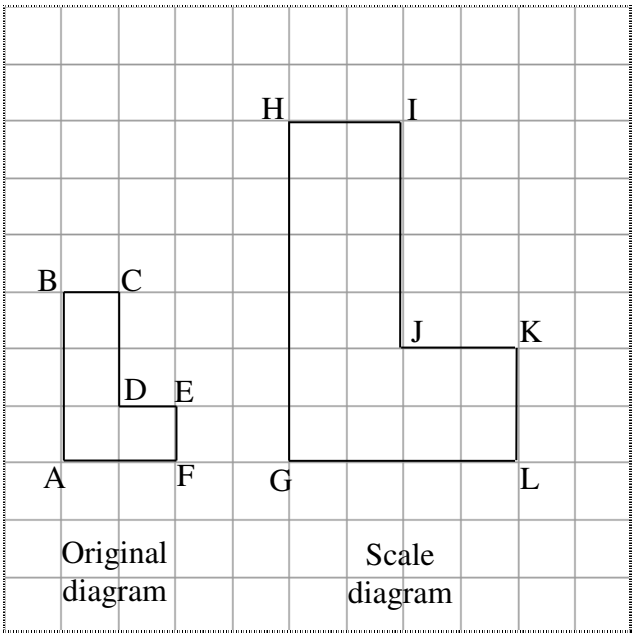
7.1 – Scale Diagrams and Enlargements

Focus: Draw and interpret scale diagrams that represent enlargements

Scale Diagram

A ***scale drawing*** represents objects that are either too large or too small to be drawn to actual size. A scale drawing of an object has the same shape as the actual object and is *similar* to it.

In a scale drawing, all the parts of the diagram are in proportion of to their actual size. Examples of scale drawings include blueprints, maps and floor plans. Every scale drawing has a **scale factor**, which is the ratio of the length of the drawing to the length of the actual object.



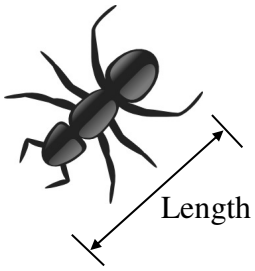
Matching lengths on the original diagram and the scale diagram are called **corresponding lengths**.

$$\textbf{Scale Factor*} = \frac{\text{scale diagram}}{\text{original diagram}}$$

*To calculate scale factor, the units you measure each diagram with must be the same.

Using Corresponding Lengths to Determine the Scale Factor

Ex. 1: Laura drew a picture of a black ant that measured 33 mm in length. If the actual length of a black ant is 12 mm, determine the scale factor of the diagram. (As a fraction and a decimal)

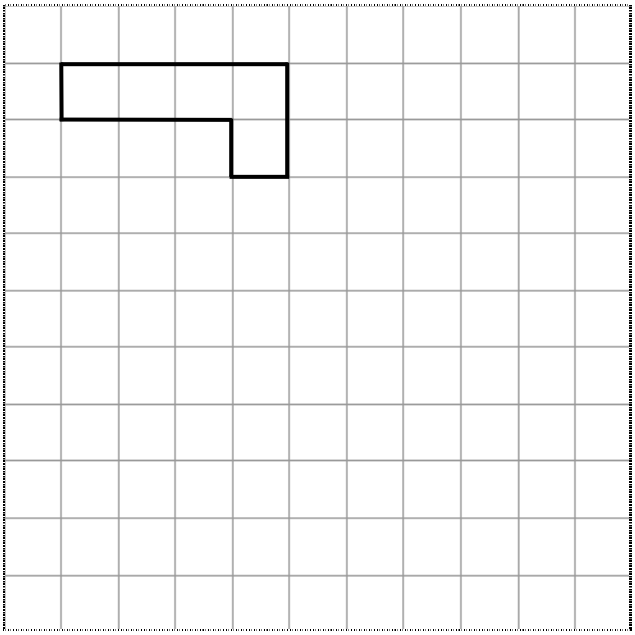


Using a Scale Factor to Determine Dimensions

Ex. 2: A painting has dimensions 30 cm by 21 cm. The painting is to be enlarged by a scale factor of $\frac{8}{3}$. Calculate the dimensions of the enlargement.

Drawing a Scale Diagram that Is an Enlargement

Ex. 3: Measure the length of each line segment in the given diagram. Draw a scale diagram with a scale factor of 2.5.



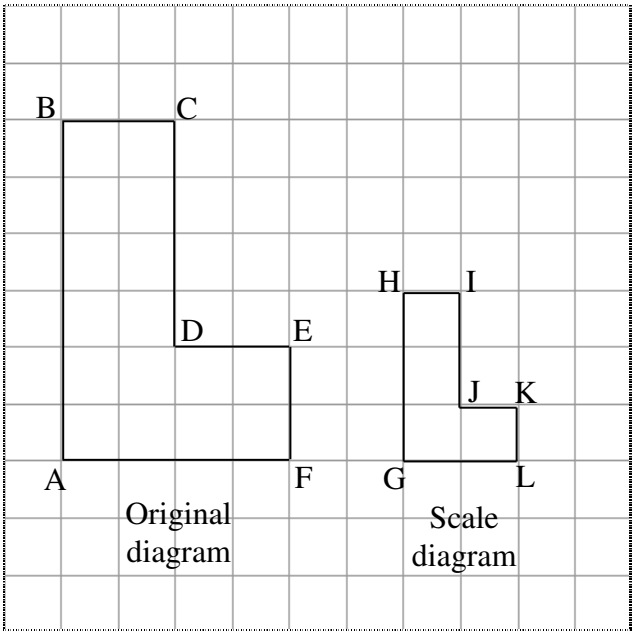
HW Assignment
Section 7.1 pg. 323 # 4 – 8, 11, 12

7.2 – Scale Diagrams and Reductions

Focus: Draw and interpret scale diagrams that represent reductions.

Scale Diagram

A scale diagram can be smaller than the original diagram.
This type of scale diagram is called a **reduction**.

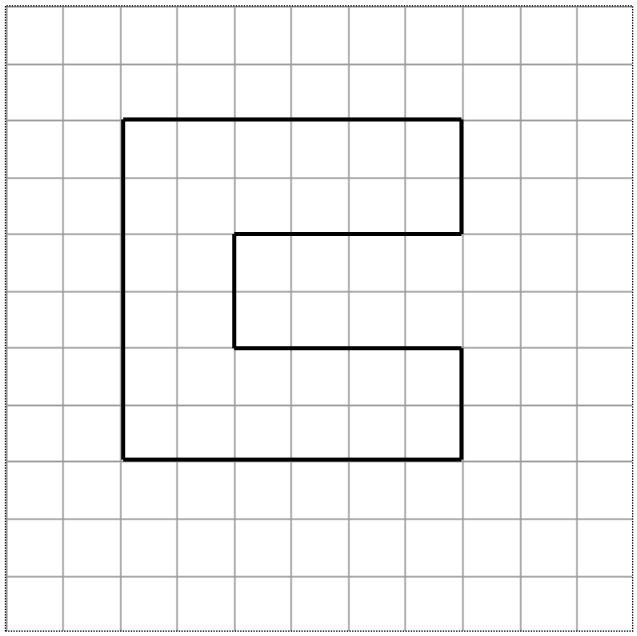


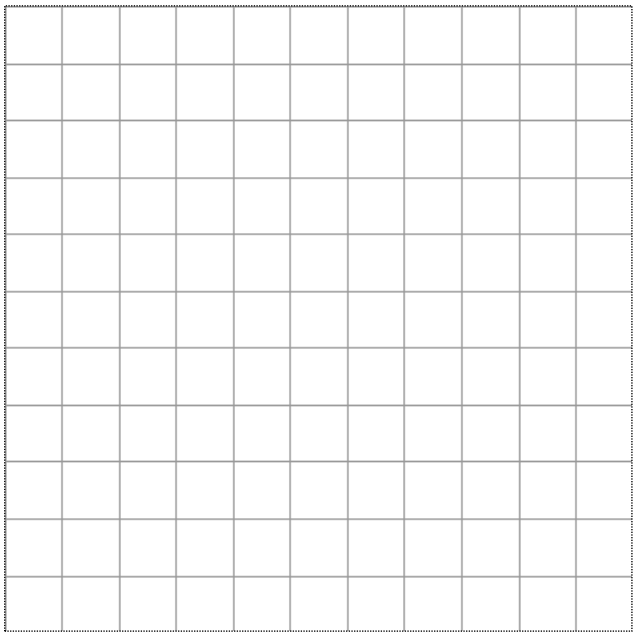
Scale Factor* $= \dfrac{\text{scale diagram}}{\text{original diagram}}$

*To calculate scale factor, the units you measure each diagram with must be the same.

Drawing a Scale Diagram that Is an Reduction

Ex. 1: Measure the length of each line segment in the given diagram.
Draw a scale diagram with a scale factor of 0.25.

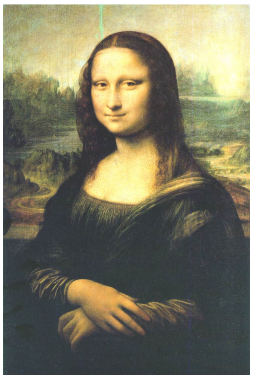




Scale factor can also be represented in a colon notation, such as 1:25. It means a scale factor of $\frac{1}{25}$.

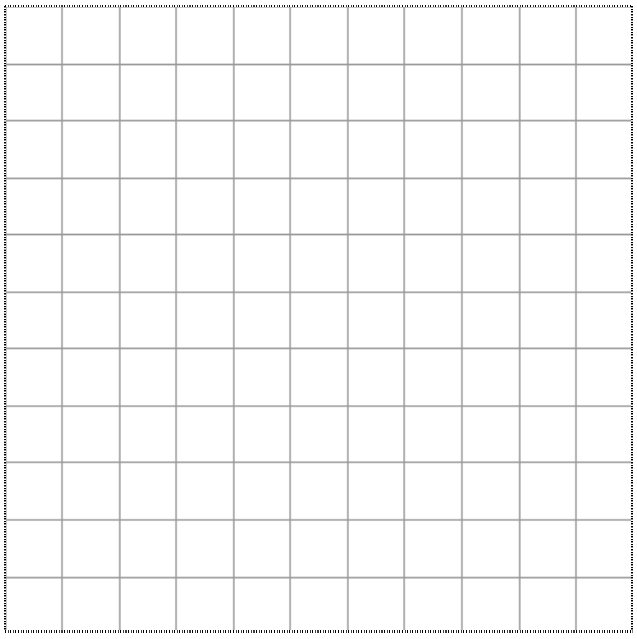
Using a Scale to Determine Lengths

Ex. 2: The Mona Lisa has approximate dimensions 75 cm by 55 cm. If we reduce the painting by a scale factor of 1:15, what are the dimensions of the reduction to the nearest hundredth of a cm?



Reduction

Ex. 3: A rectangular auditorium has dimensions of length 22 m and width 14 m. Represent the auditorium with a scale drawing of 1 : 400. Label your drawing in cm.



Reflection: When you are given a scale factor, how do you know if it is an enlargement or a reduction?

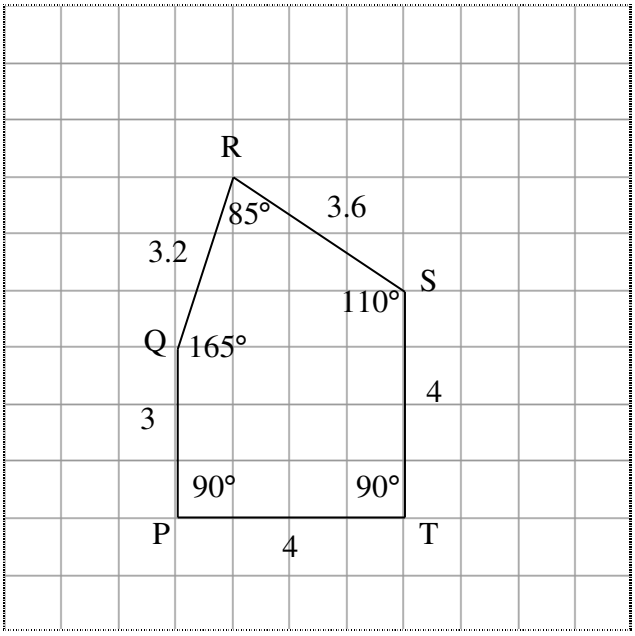
HW Assignment
Section 7.2 pg. 329 # 4 – 6, 8 – 15

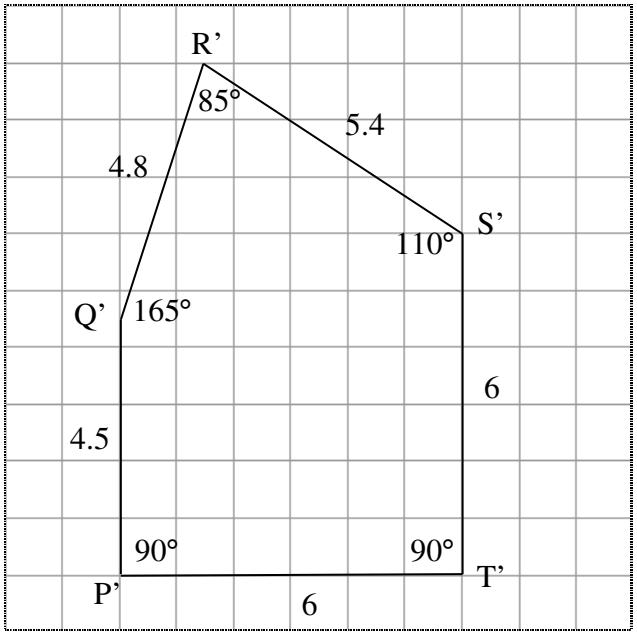
7.3 – Similar Polygons

Focus: Recognize and draw similar polygons, then use their properties to solve problems.

Similar Polygons

When one polygon is an enlargement or a reduction of another polygon, we say the polygons are similar. Similar polygons have the same shape, but not necessarily the same size.





Matching angles are **corresponding angles**. Matching sides are **corresponding sides**.

Corresponding Sides			Corresponding Angles	
PQ =	P'Q' =	$\frac{P'Q'}{PQ}$ =	∠P =	∠P' =
QR =	Q'R' =	$\frac{Q'R'}{QR}$ =	∠Q =	∠Q' =
RS =	R'S' =	$\frac{R'S'}{RS}$ =	∠R =	∠R' =
ST =	S'T' =	$\frac{S'T'}{ST}$ =	∠S =	∠S' =
TP =	T'P' =	$\frac{T'P'}{TP}$ =	∠T =	∠T' =

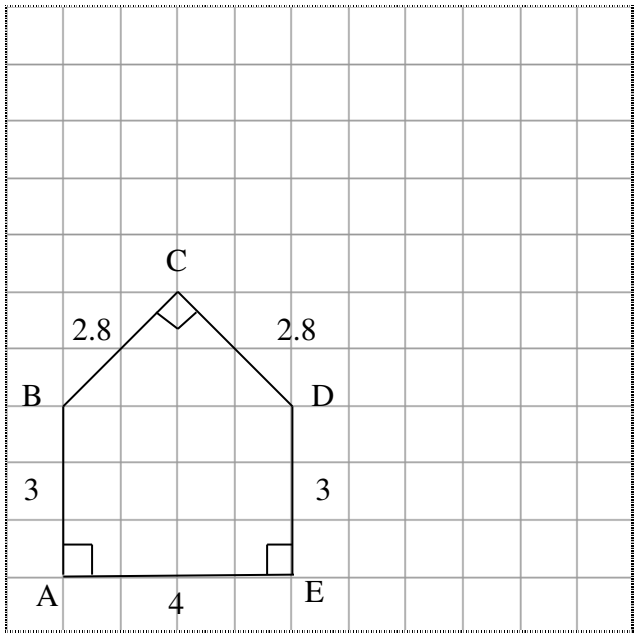
Properties of Similar Polygons

When two polygons are similar:

- their corresponding angles are equal, **and**
- their corresponding sides are proportional

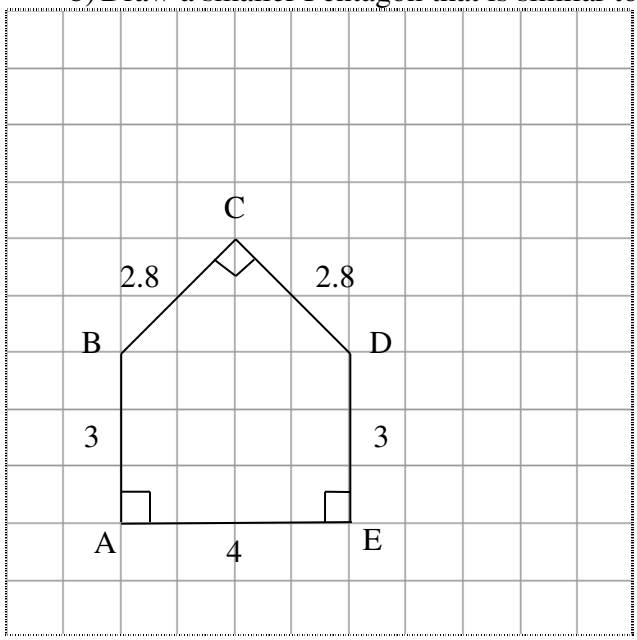
Drawing a Polygon Similar to a Given Polygon

Ex. 1: a) Draw a larger Pentagon that is similar to this pentagon. (Choose a scale factor, such as 2)



Explain why the polygons are similar.

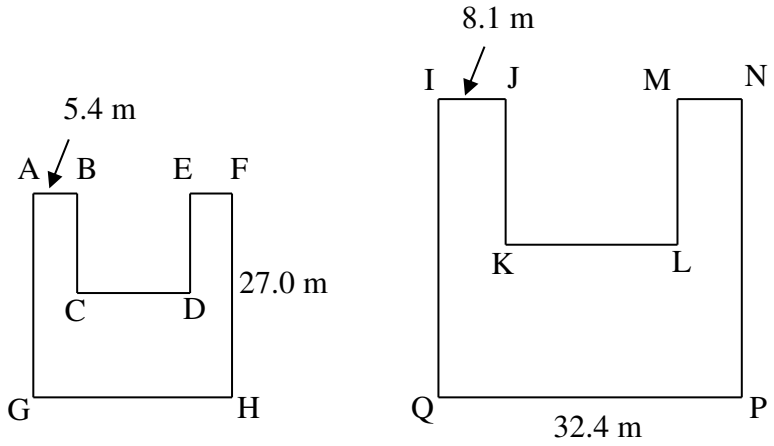
b) Draw a smaller Pentagon that is similar to this pentagon. (Choose a scale factor, such as 0.5)



Explain why the polygons are similar.

Solving Problems Using the Properties of Similar Polygons

Ex. 2.: These two octagonal garden plots are similar.



a) Calculate the length of GH.

b) Calculate the length of NP.

HW Assignment
Section 7.3 pg. 341 # 4 – 7, 9 – 12, 14

7.4 – Similar Triangles

Focus: Use the properties of similar triangles to solve problems.

Similar Triangles

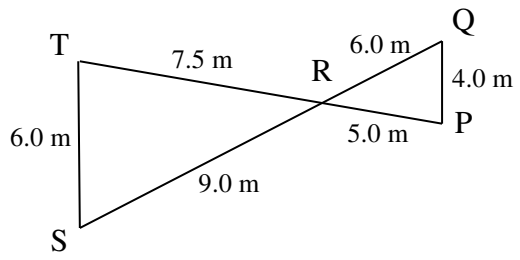
A triangle is a special polygon.

When two triangles are similar:

- their corresponding angles are equal, **or**
- their corresponding sides are proportional

Using Corresponding Sides to Name Similar Triangles

Ex. 1: Identify the similar triangles. Justify your answer

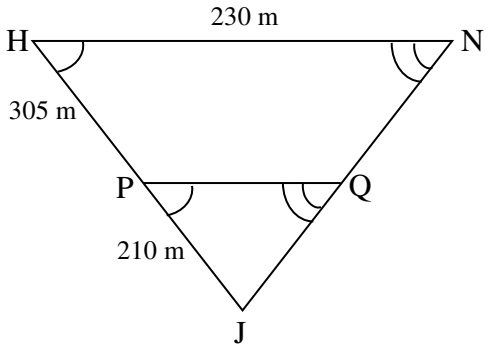


Using Similar Triangles to Determine a Length

Ex. 2. At a certain time of day, a person who is 1.8 m tall has a shadow 1.3 long. At the same time, the shadow of a totem pole is 6 m long. The sun's rays intersect the ground at equal angles. How tall is the totem pole, to the nearest tenth of a metre?

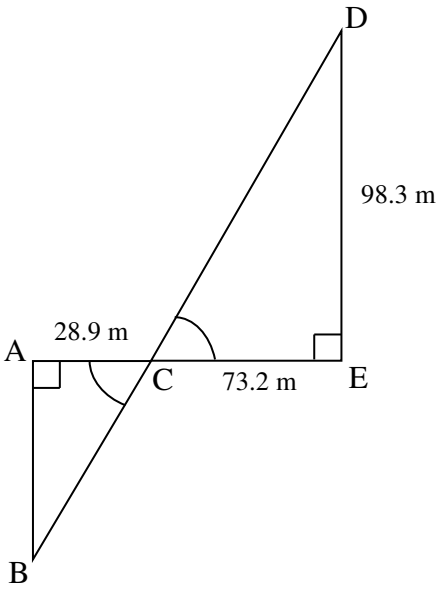
Using Overlapping Similar Triangles to Determine a Length

Ex. 3: A surveyor wants to determine the width of a lake at two points on opposite sides of the lake. She measures distances and angles on land, then sketch this diagram. How can the surveyor determine the length HN to the nearest tenth?



Using Triangles Meeting at a Vertex to Determine a Length

Ex. 4: A surveyor used this scale diagram to determine the width of a river. The measurements he made and the equal angles are shown. What is the width, AB, to the nearest tenth of a metre?

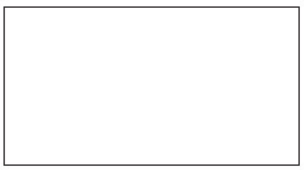


HW Assignment
Section 7.4 pg. 349 # 4 – 7, 9 – 15
Quiz next class on 7.1 to 7.4

Mid-Unit Review

7.1 **1.** A photo of a gymnast is to be enlarged. The dimensions of the photo are 15 cm by 10 cm. What are the dimensions of the enlargement with a scale factor of $\frac{7}{5}$?

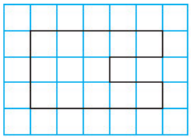
2. A computer chip has dimensions 15 mm by 8 mm. Here is a scale drawing of the chip.



a) Determine the scale factor of the diagram.

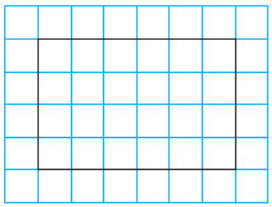
b) Draw a scale diagram of the chip with a scale factor of 8.

7.2 **3.** a) Copy this polygon on 1-cm grid paper.

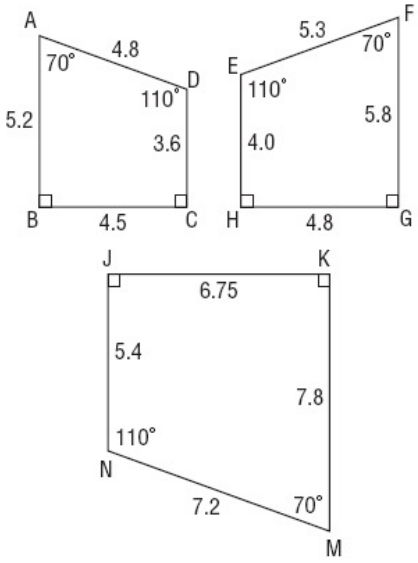


b) Draw a scale diagram of the polygon with a scale factor of $\frac{3}{5}$. Show any calculations you made.

4. This top view of a swimming pool is drawn on 0.5-cm grid paper. The dimensions of the pool are 60 m by 40 m. Determine the scale factor of the reduction as a fraction or a decimal.



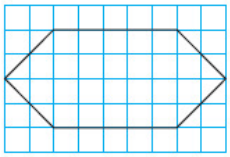
7.3 **5.** These quadrilaterals have corresponding angles equal.



a) Are any of these quadrilaterals similar? Justify your answer.

b) Choose one quadrilateral. Draw a similar quadrilateral. How do you know the quadrilaterals are similar?

6. A window has the shape of a hexagon.



Draw a hexagon that is similar to this hexagon. Explain how you know the hexagons are similar.

7.4 **7.** A tree casts a shadow 8 m long. At the same time a 2-m wall casts a shadow 1.6 m long.

a) Sketch a diagram.

b) What is the height of the tree?

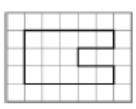
Answers:

1. 21 cm by 14 cm

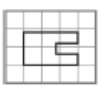
2. a) The diagram is an enlargement with a scale factor of 3.

b) The scale diagram is a 12-cm by 6.4-cm rectangle.

3. a)



b)

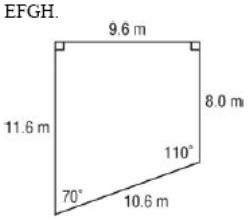


4. $\frac{1}{2000}$

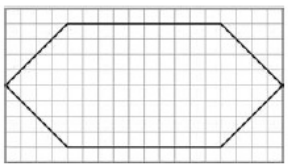
5. a) Quadrilateral ABCD ~ quadrilateral MKJN; the corresponding sides are proportional:

$\frac{AB}{MK} = \frac{BC}{KJ} = \frac{CD}{JN} = \frac{DA}{NM} = \frac{2}{3}$

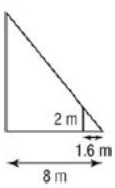
b) Answers will vary. For example:
This quadrilateral is similar to quadrilateral EFGH.



6. The length of each side of this hexagon is 2 times the length of the corresponding side in the original hexagon and the corresponding angles are all equal.



7. a)



b) The height of the tree is 10 m.

7.5 – Reflections and Line Symmetry

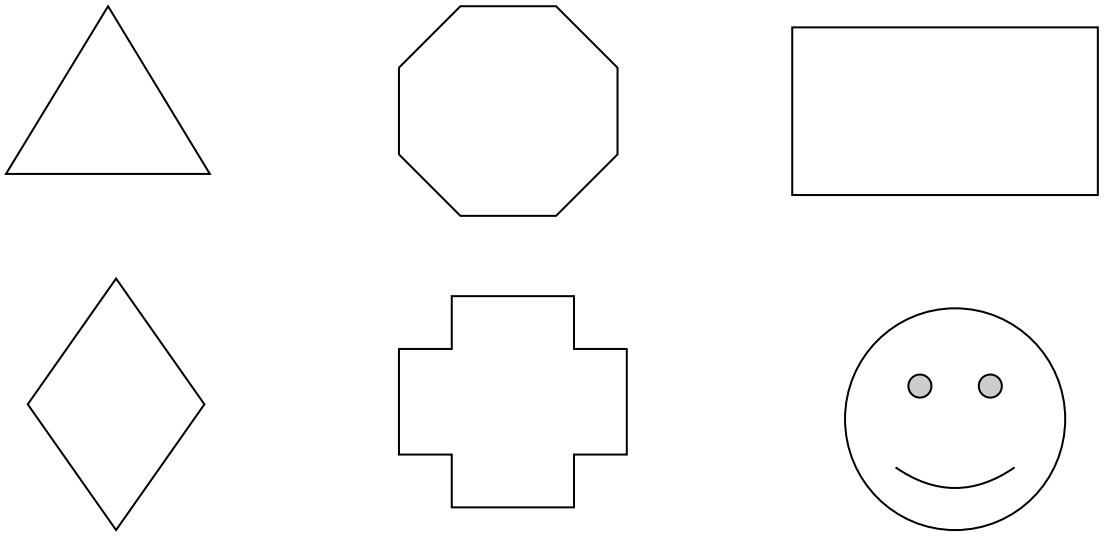
Focus: Draw and classify shapes with line symmetry.

Line of Symmetry

Line of symmetry is a line that divides a figure into two reflected parts. Sometime it's called a line of reflection or axis of symmetry. A figure may have one or more lines of symmetry, or it may have none. Line of symmetry can be vertical, horizontal or oblique (slanted).

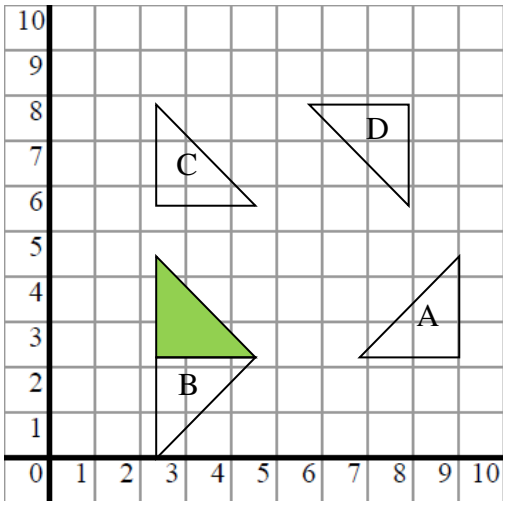
Determine Lines of Symmetry

Ex. 1: Identify the lines of symmetry in each.



Identifying Shapes Related by a Line of Reflection

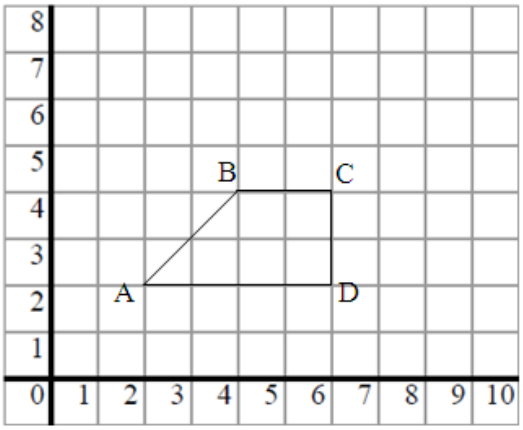
Ex. 2. Identify the triangles that are related to the green triangle by a line of reflection. Describe the position of each line of symmetry.



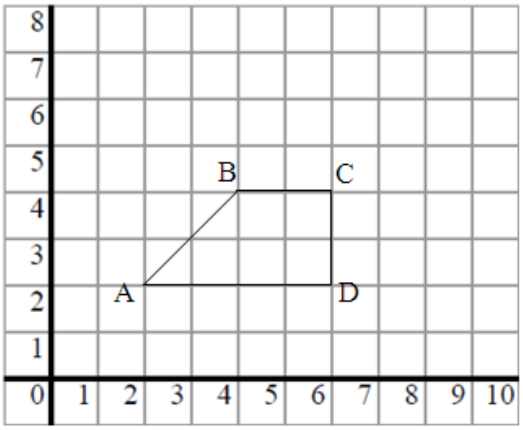
Completing a Shape Given its Line of Symmetry

Ex. 3: Quadrilateral ABCD is part of a larger shape. Draw the image of ABCD after each reflection. Write the coordinates of the larger shape formed by ABCD and its reflection image. Describe the larger shape and its symmetry.

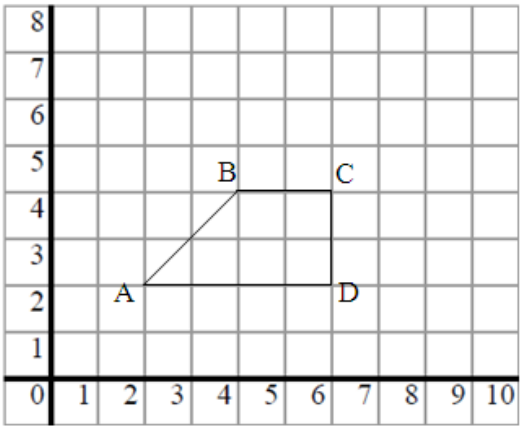
a) a reflection in the horizontal line through 2 on the *y*-axis.



b) a reflection in the horizontal line through 6 on the *x*-axis.

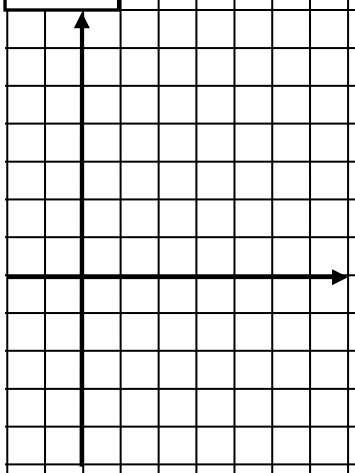


c) a reflection in an line through (0,0) and (6,6).

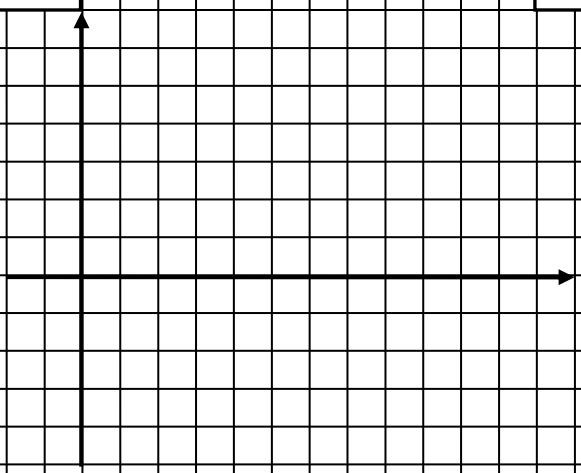


HW Assignment
Section 7.5 pg. 357 # 3, 5, 6, 8 – 10

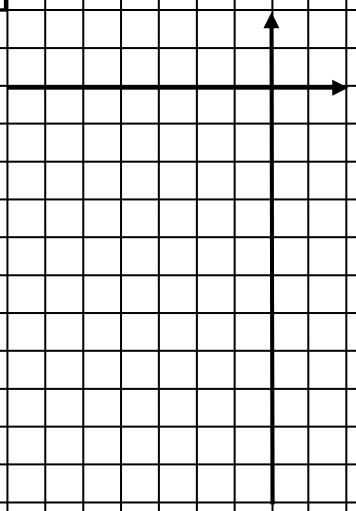
5. a)



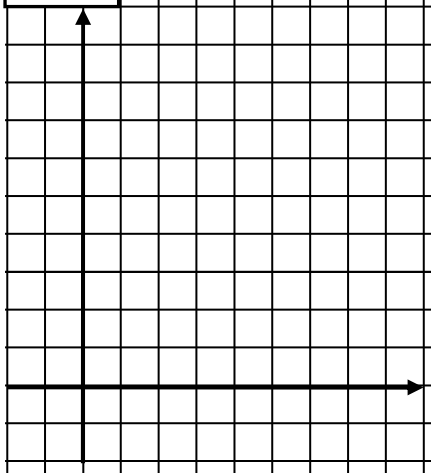
5. b)



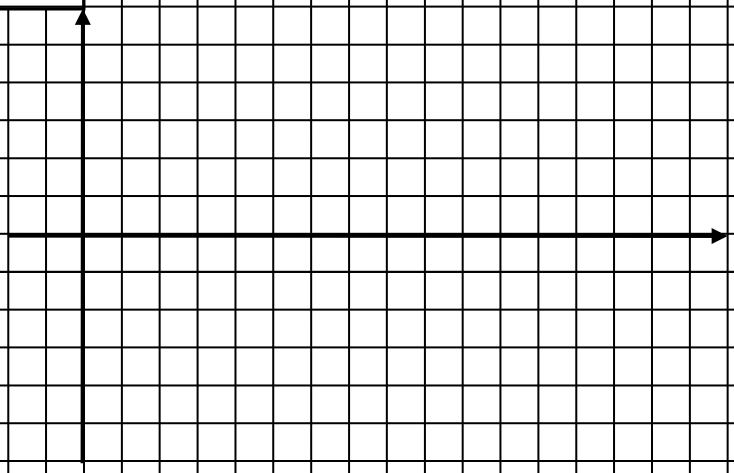
5. c)



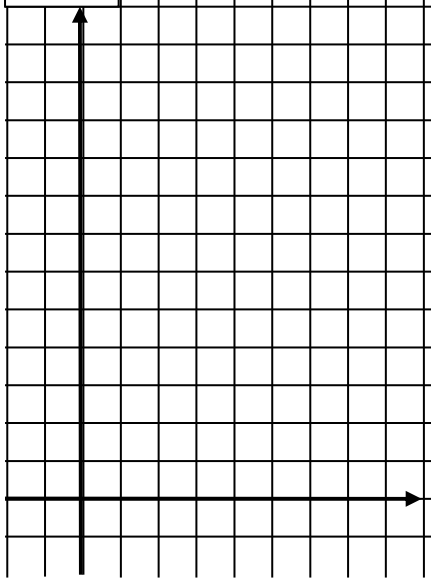
8. a)



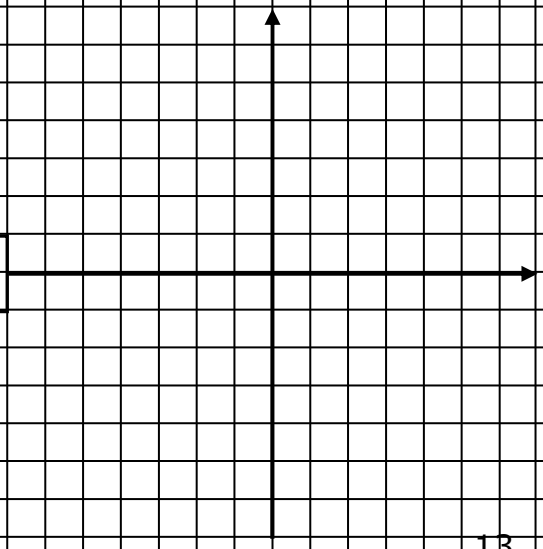
8. b)



8. c)



9.



7.6 – Rotations and Rotational Symmetry

Focus: Draw and classify shapes with rotational symmetry.

Rotational Symmetry

Rotational symmetry occurs when a shape or design can be turned about its centre of rotation so that it fits onto its outline *more than once* in a complete turn.

Order of Rotation

The number of times a shape or design fits onto itself in one complete turn. For any shape the order is at least one.

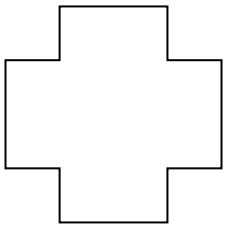
Angle of Rotation

Angle of rotation is the minimum measure of the angle needed to turn a shape onto itself. It may be measured in degrees or fractions of a turn.

$$\text{angle of rotation symmetry} = \frac{360°}{\text{the order of rotation}}$$

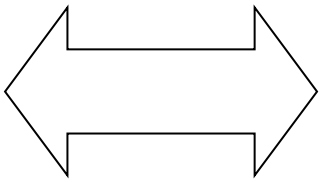
Identifying Shapes with Rotational Symmetry

Ex. 1: Determine which have rotational symmetry.
State the order of rotation and the angle of rotation symmetry.

a) 

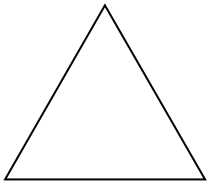
the order of rotation :

the angle of rotation symmetry:

b) 

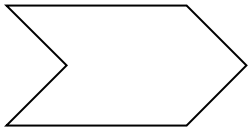
the order of rotation :

the angle of rotation symmetry:

c) 

the order of rotation :

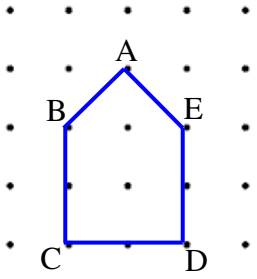
the angle of rotation symmetry:

d) 

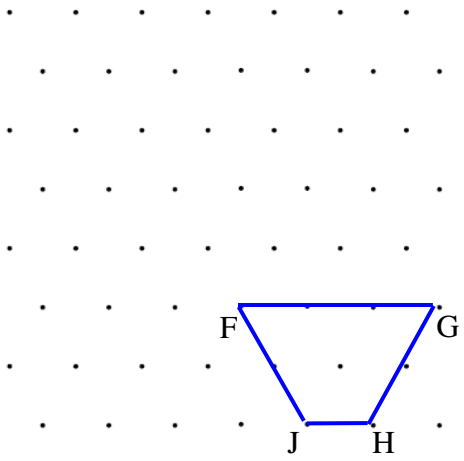
the order of rotation :
the angle of rotation symmetry:

Drawing Rotation Images

Ex. 2: a) Rotate pentagon ABCDE 90° clockwise about vertex E. Draw the rotation image.



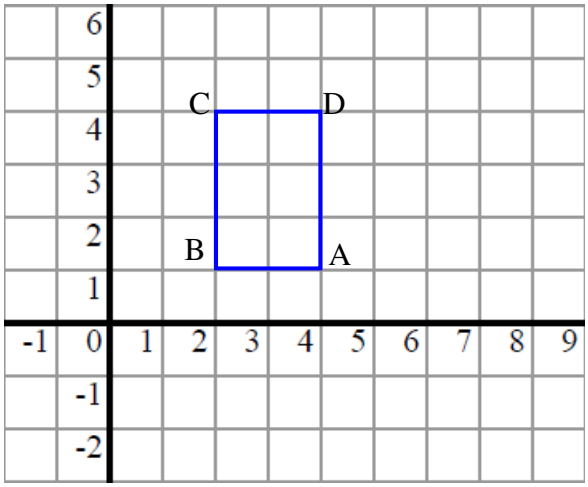
b) Rotate trapezoid FGHJ 120° counter-clockwise about vertex F. Draw the rotation image.



Identifying Symmetry after Rotations

Ex. 3: Rotate rectangle ABCD.

a) 90° clockwise about the vertex A; 180° clockwise about the vertex A; 270° clockwise about the vertex A.



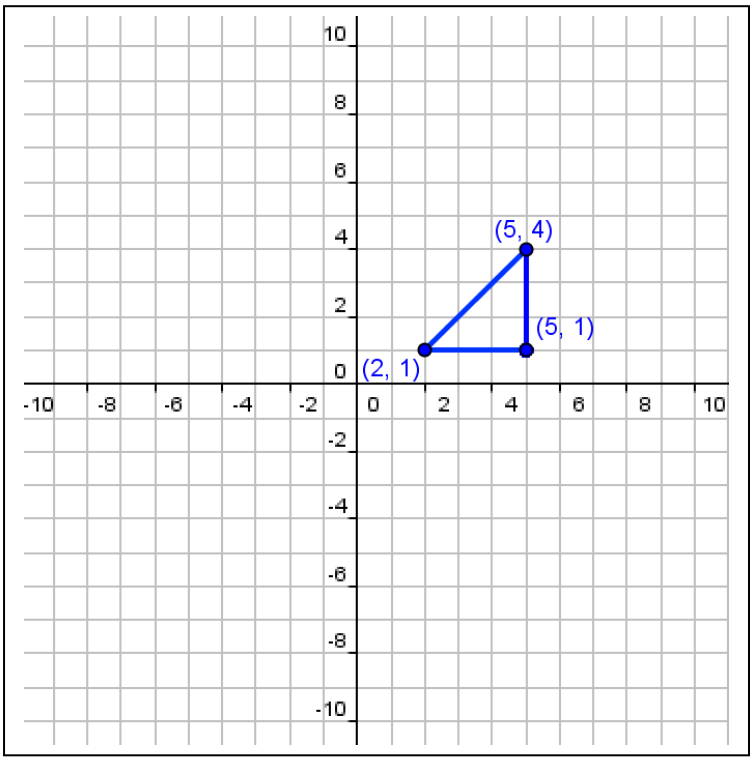
b) Look at the shape formed by the rectangle and all its images. Identify any rotational symmetry in this shape.

Ex. 4:

a) Rotate the triangle 270° clockwise about the point (2, 1)

b) Rotate the triangle 180° clockwise about the point (5, 1)

c) 180° counter clockwise about the origin.



Ex. 5: What shapes do not have rotational symmetry? Draw an example.

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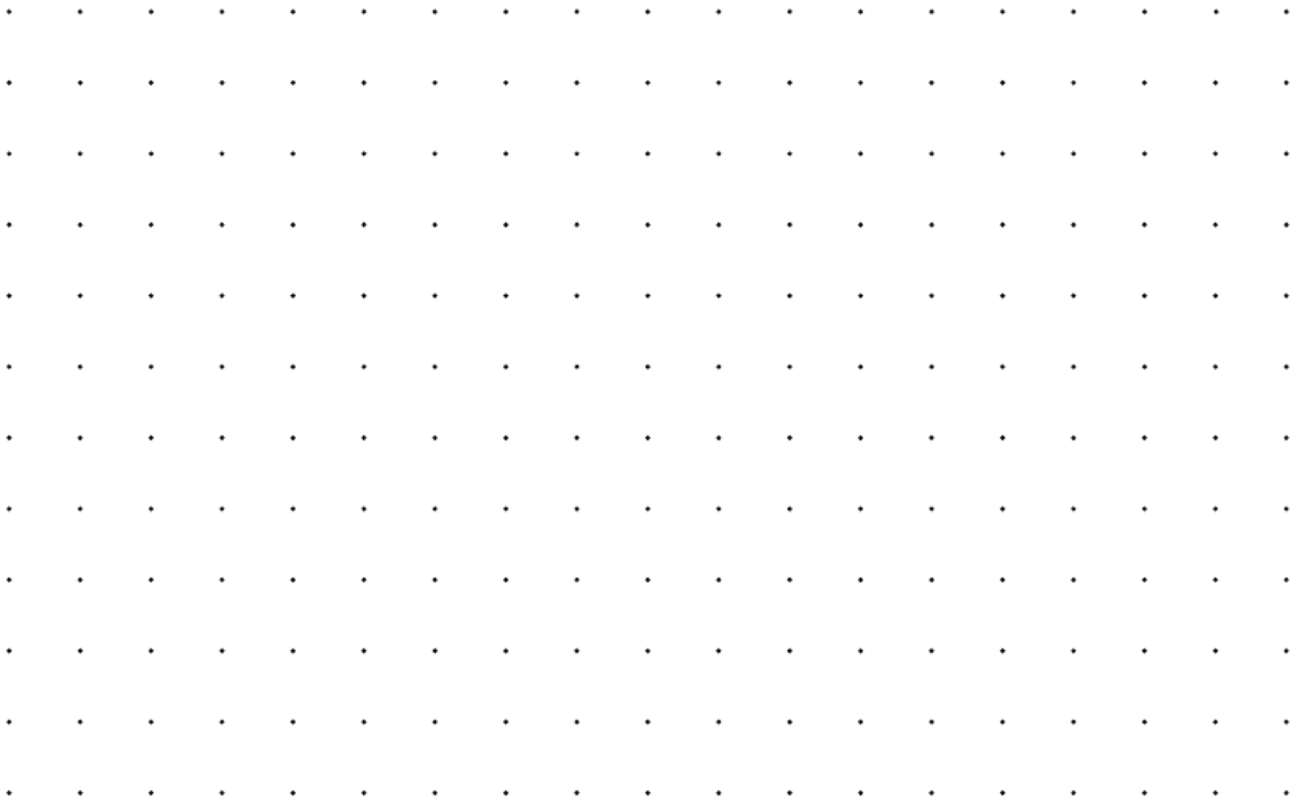
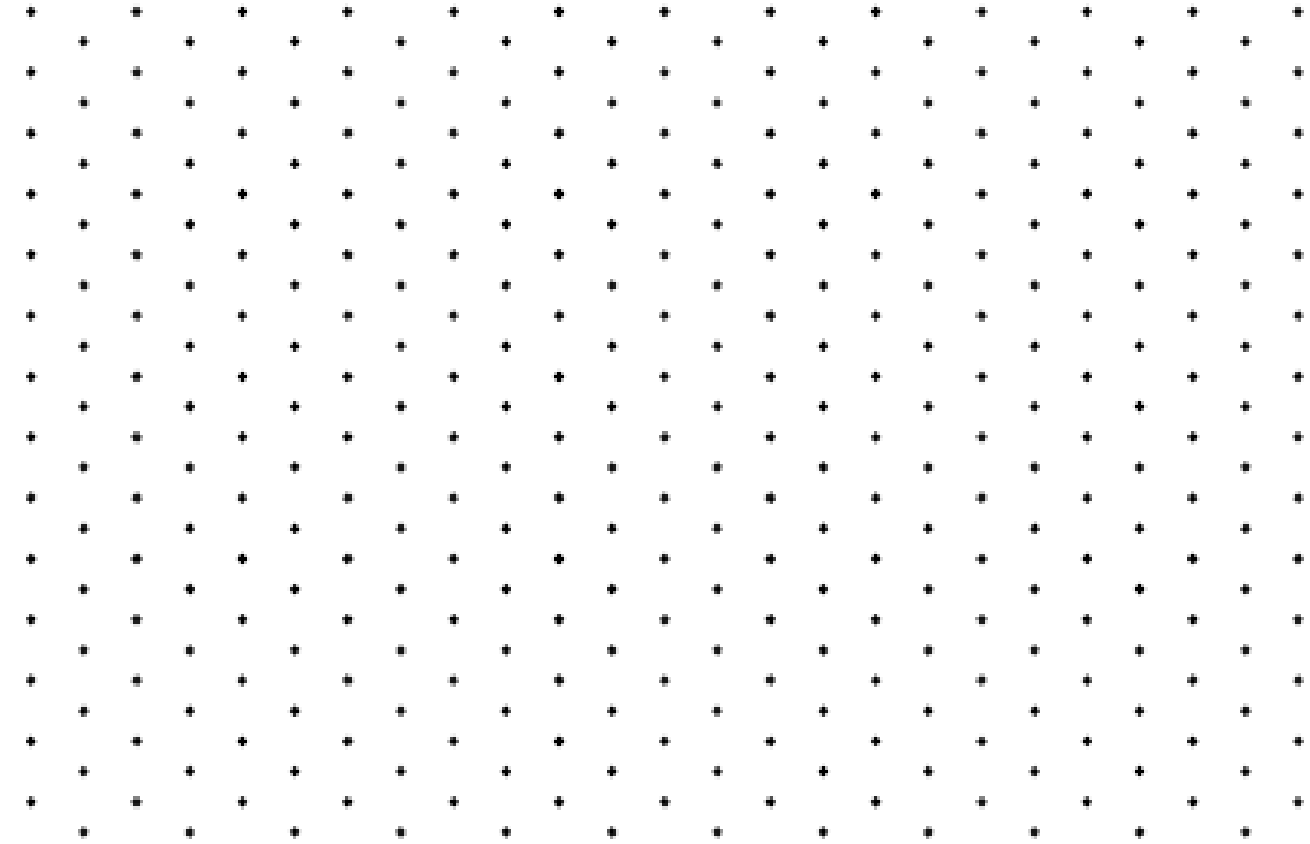
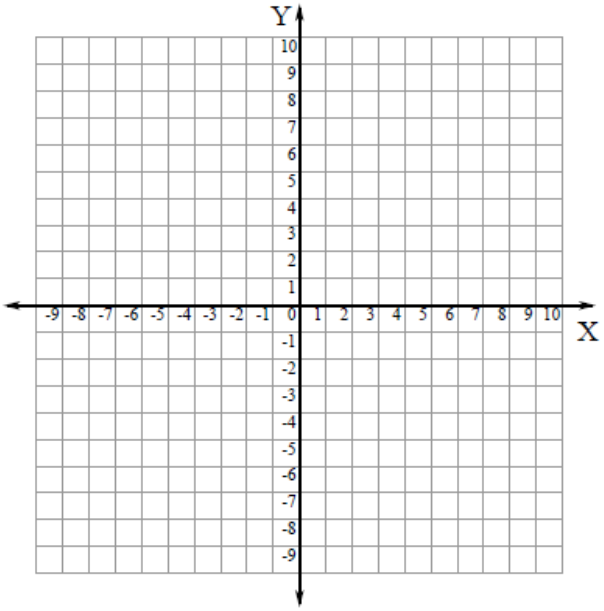
9, 13ab



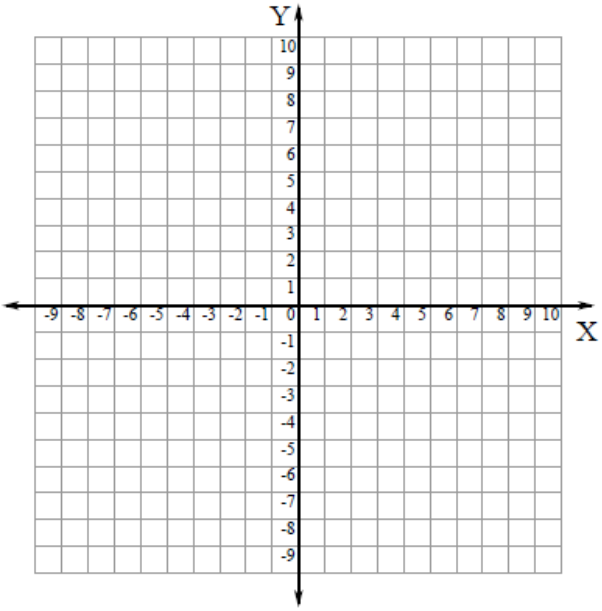
10, 13c



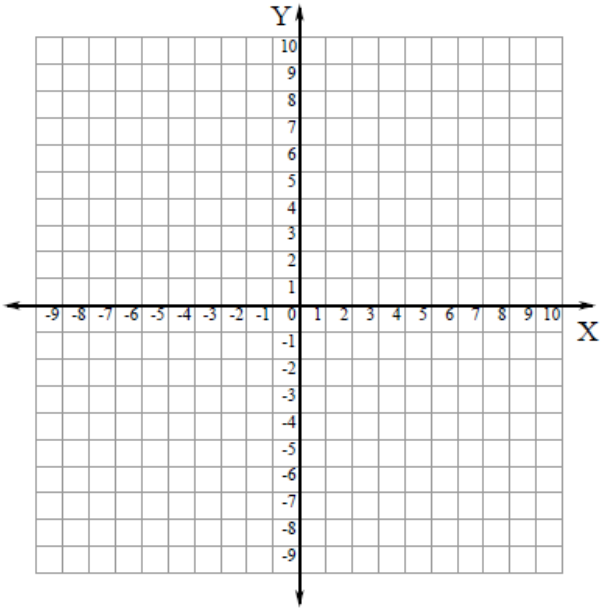
12.



14.



15.



7.7 – Identifying Types of Symmetry on the Cartesian Plane

Focus: Identify and classify line and rotational symmetry.

Transformations

A *transformation* is a general term for four specific ways to manipulate the shape of a point, a line, or shape. The original shape of the object is called the pre-image and the final shape and position of the object is the image under the transformation.

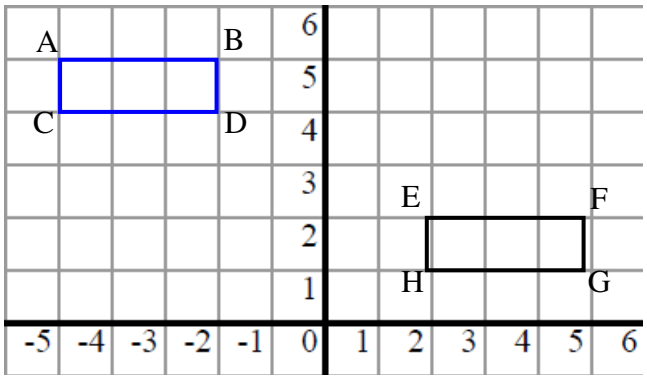
The types of transformations in mathematics are:

1) **Reflection**
2) **Rotation**
3) **Dilation** (enlarging or reducing the original shape)
4) **Translation** (moving the original shape without turning or flipping it).

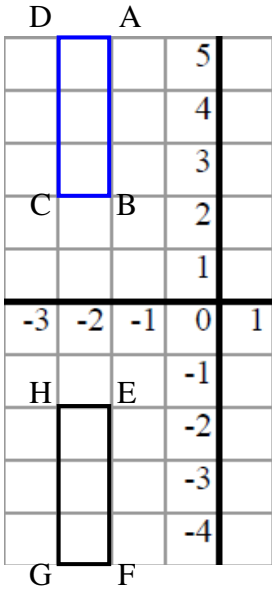
Determine whether Shapes Are Related by Symmetry

Ex. 1: For each pair of rectangles ABCD and EFGH, determine whether they are related by symmetry.

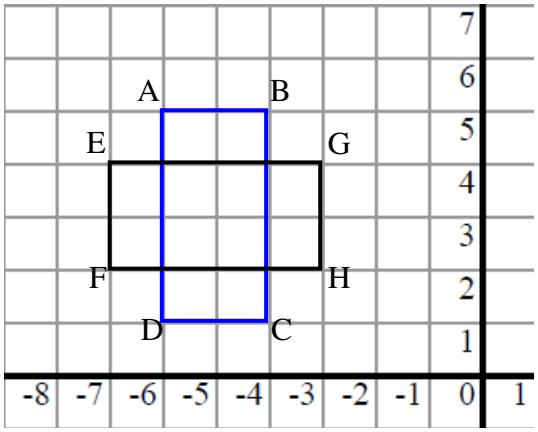
a)



b)



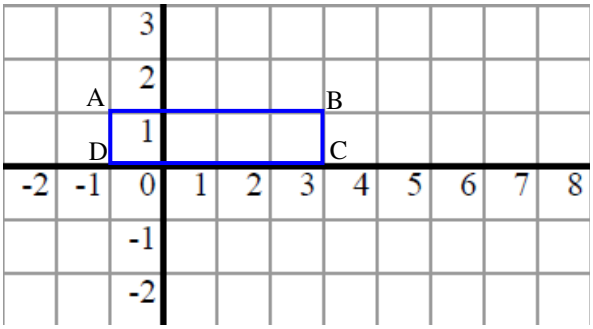
c)



Identifying Symmetry in a Shape and its Transformation Image

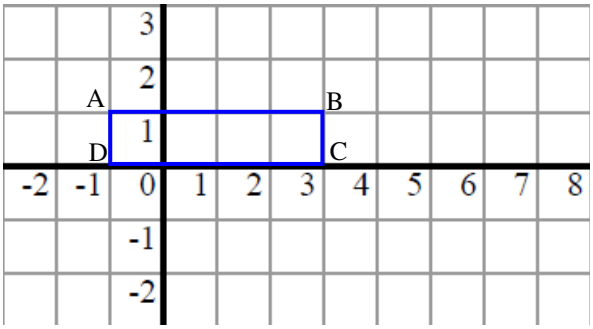
Ex. 2: Draw the image of rectangle ABCD after each transformation. Write the coordinates of each vertex and its image. Identify and describe the type of symmetry that results.

a) a rotation of 180° about the origin



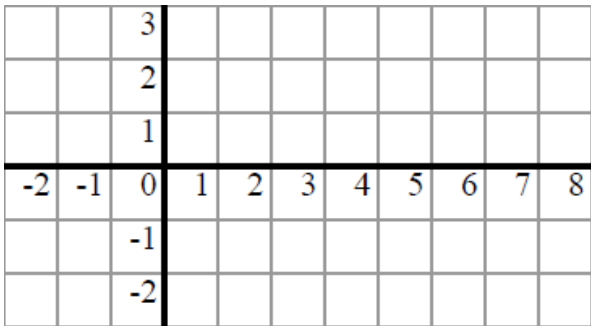
Point	Image

b) a reflection in the *x*-axis



Point	Image

c) a translation 4 units right and 1 unit down

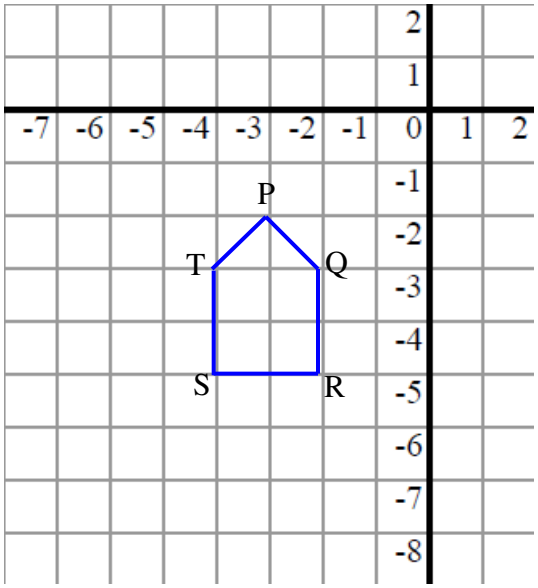


Point	Image

Identifying Symmetry in Shapes and their Transformation Images

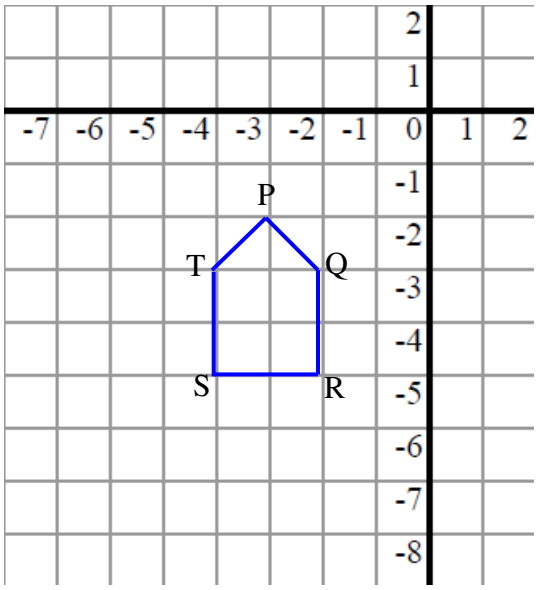
Ex. 3. Draw the image of pentagon PQRST after each translation below. Label the vertices of the pentagon and its image, and list their coordinates. If each diagram has symmetry, describe it. If each diagram does not have symmetry, explain how you know.

a) a translation L2 (move 2 units to the left)



Point	Image

b) a translation L2, D3 (move 2 units to the left and 3 units down)



Point	Image

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Study Guide

Scale Diagrams

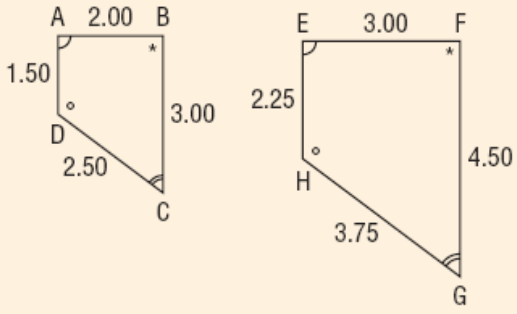
For an enlargement or reduction, the scale factor is: $\frac{\text{Length on scale diagram}}{\text{Length on original diagram}}$

An enlargement has a scale factor > 1. A reduction has a scale factor < 1.

Similar Polygons

Similar polygons are related by an enlargement or a reduction. When two polygons are similar:

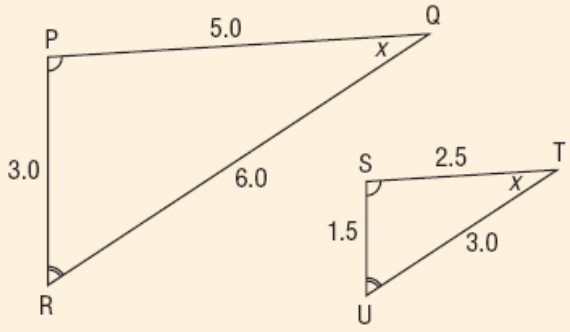
- their corresponding angles are equal:
  $\angle A = \angle E$; $\angle B = \angle F$; $\angle C = \angle G$; $\angle D = \angle H$
  and
- their corresponding sides are proportional:
  $\frac{AB}{EF} = \frac{BC}{FG} = \frac{CD}{GH} = \frac{DA}{HE}$
  Any of the ratios $\frac{AB}{EF}$, $\frac{BC}{FG}$, $\frac{CD}{GH}$, and $\frac{DA}{HE}$ is the scale factor.



Similar Triangles

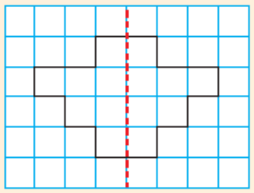
When we check whether two triangles are similar:

- their corresponding angles must be equal:
  $\angle P = \angle S$ and $\angle Q = \angle T$ and $\angle R = \angle U$
  or
- their corresponding sides must be proportional:
  $\frac{PQ}{ST} = \frac{QR}{TU} = \frac{PR}{SU}$
  Any of the ratios $\frac{PQ}{ST}$, $\frac{QR}{TU}$, and $\frac{PR}{SU}$ is the scale factor.



Line Symmetry

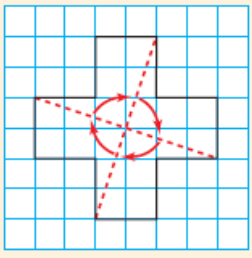
A shape has line symmetry when a line divides the shape into two congruent parts so that one part is the image of the other part after a reflection in the line of symmetry.



Rotational Symmetry

A shape has rotational symmetry when it coincides with itself after a rotation of less than 360° about its centre. The number of times the shape coincides with itself is the order of rotation.

The angle of rotation symmetry $= \frac{360°}{\text{the order of rotation}}$



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