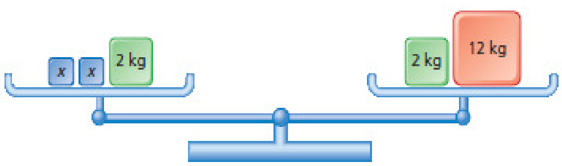
7.1 Developing Systems of Linear Equations

LESSON FOCUS

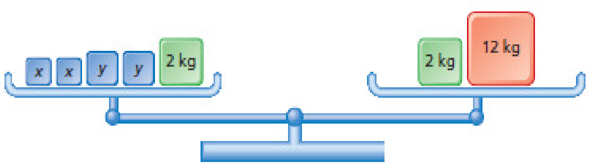
Model a situation using a system of linear equations.

Make Connections

Which linear equation relates the masses on these balance scales?



Which linear equation relates the masses on these balance scales?



How are the two equations the same? How are they different?

What do you know about the number of solutions for each equation?

A school district has buses that carry 12 passengers and buses that carry 24 passengers. The total passenger capacity is 780. There are 20 more small buses than large buses.

To determine how many of each type of bus there are, we can write two equations to model the situation.
We first identify the unknown quantities.
There are some small buses but we don't know how many.
Let s represent the number of small buses.

There are some large buses but we don't know how many.
Let l represent the number of large buses.
The total passenger capacity is 780.
Each small bus carries 12 people and each large bus carries 24 people.
So, this equation represents the total capacity: $12s + 24l = 780$
There are 20 more small buses than large buses.
So, this equation relates the numbers of buses: $s = l + 20$

These two linear equations model the situation:
$12s + 24l = 780$
$s = l + 20$

These two equations form a **system of linear equations** in two variables, s and l.
A system of linear equations is often referred to as a **linear system.**
A *solution* of a linear system is a pair of values of s and l that satisfy both equations.

Suppose you are told that there are 35 small buses and 15 large buses.
To verify that this is the solution, we compare these data with the given situation.
The difference in the numbers of small and large buses is: $35 - 15 = 20$
Calculate the total capacity of 35 small buses and 15 large buses.

$$\begin{aligned}\text{Total capacity} &= 35(12) + 15(24)\\ &= 420 + 360\\ &= 780\end{aligned}$$

The difference in the numbers of small and large buses is 20 and the total passenger capacity is 780. This agrees with the given data, so the solution is correct.

We can also verify the solution by substituting the known values of s and l into the equations.
In each equation, substitute: $s = 35$ and $l = 15$

$12s + 24l = 780$

$$\begin{aligned}\text{L.S.} &= 12s + 24l\\ &= 12(35) + 24(15)\\ &= 420 + 360\\ &= 780\\ &= \text{R.S.}\end{aligned}$$

$s = l + 20$

$$\begin{aligned}\text{L.S.} &= s\\ &= 35\end{aligned}$$

$$\begin{aligned}\text{R. S.} &= l + 20\\ &= 15 + 20\\ &= 35\\ &= \text{L.S.}\end{aligned}$$

For each equation, the left side is equal to the right side. Since $s = 35$ and $l = 15$ satisfy each equation, these numbers are the solution of the linear system.

Example 1: Using a Diagram to Model a Situation

a) Create a linear system to model this situation:
The perimeter of a Nunavut flag is 16 ft.
Its length is 2 ft. longer than its width.

b) Denise has determined that the Nunavut flag is 5 ft. long and 3 ft. wide.
Use the linear system from part a to verify that Denise is correct.

CHECK YOUR UNDERSTANDING

1. a) Create a linear system to model this situation:
The stage at the Lyle Victor Albert Centre in Bonnyville, Alberta, is rectangular.
Its perimeter is 158 ft.
The width of the stage is 31 ft. less than the length.

b) Sebi has determined that the stage is 55 ft. long and 24 ft. wide. Use the linear system from part a to verify that Sebi is correct.

[Answer: a) $2l + 2w = 158$; $w = l - 31$]

Example 2: Using a Table to Create a Linear System to Model a Situation

a) Create a linear system to model this situation:
In Calgary, a school raised $195 by collecting 3000 items for recycling.
The school received 5¢ for each pop can and 20¢ for each large plastic bottle.

b) The school collected 2700 pop cans and 300 plastic bottles.
Use the linear system to verify these numbers.

a)

	Refund per Item ($)	Number of Items	Money Raised ($)
Can			
Bottle			
Total			

CHECK YOUR UNDERSTANDING

2. a) Create a linear system to model this situation:
A school raised $140 by collecting 2000 cans and glass bottles for recycling.
The school received 5¢ for a can and 10¢ for a bottle.

b) The school collected 1200 cans and 800 bottles.
Use the linear system to verify these numbers.

[Answer: a) $0.05c + 0.10b = 140$; $c + b = 2000$]

For this situation:

A store display had packages of 8 batteries and packages of 4 batteries.
The total number of batteries was 320.
There were 1.5 times as many packages of 4 batteries as packages of 8 batteries.

Cary wrote this linear system:
$8e + 4f = 320$
$1.5f = e$
where e represents the number of packages of 8 batteries and f represents the number of packages of 4 batteries.

Cary's classmate, Devon, said that the solution of the linear system was:
There are 30 packages of 8 batteries and 20 packages of 4 batteries.

To verify the solution, in each equation Cary substituted: $e = 30$ and $f = 20$

$8e + 4f = 320$	$1.5f = e$	
L.S. $= 8e + 4f$	L.S. $= 1.5f$	R.S. $= e$
$= 8(30) + 4(20)$	$= 1.5(20)$	$= 30$
$= 240 + 80$	$= 30$	$=$ L.S.
$= 320$		
$=$ R.S.		

Cary said that since the left side is equal to the right side for each equation, the solution is correct.

But when Devon used the problem to verify the solution, she realized that there should be more packages of 4 batteries than packages of 8 batteries, so the solution was wrong.

This illustrates that it is better to consider the given data to verify a solution rather than substitute in the equations. There could be an error in the equations that were written to represent the situation.

What are the correct equations for this situation?

Example 3: Relating a Linear System to a Problem

A store sells wheels for roller skates in packages of 4 and wheels for inline skates in packages of 8.

Create a situation about wheels that can be modelled by the linear system below. Explain the meaning of each variable. Write a related problem.

$8i + 4r = 440$

$i + r = 80$

CHECK YOUR UNDERSTANDING

3. A bicycle has 2 wheels and a tricycle has 3 wheels.

Create a situation about wheels that can be modelled by the linear system below. Explain the meaning of each variable. Write a related problem.

$2b + 3t = 100$

$b + t = 40$

[Sample Answer: A possible problem is: There are 40 bicycles and tricycles in a department store. The total number of wheels on all bicycles and tricycles in the store is 100. How many bicycles and how many tricycles are in the store?]

Homework: Page 401 #4–6, 8–12

7.2 Solving a System of Linear Equations Graphically

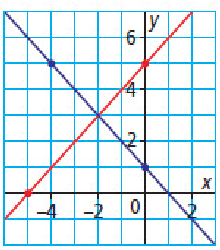
LESSON FOCUS

Use the graphs of the equations of a linear system to estimate its solution.

The town of Kelvington, Saskatchewan, erected 6 large hockey cards by a highway to celebrate the town's famous hockey players. One player is Wendel Clark.

Make Connections

Two equations in a linear system are graphed on the same grid.



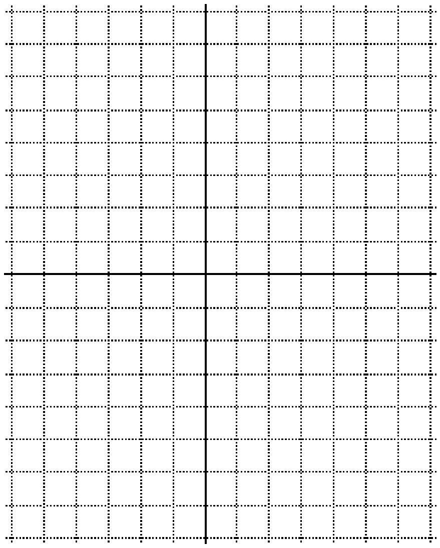
What are the equations of the graphs? Explain your reasoning.
What are the coordinates of the point of intersection of the two lines?
Explain why these coordinates are the solution of the linear system.

Example 1: Solving a Linear System by Graphing

Solve this linear system.

$x + y = 8$

$3x - 2y = 14$



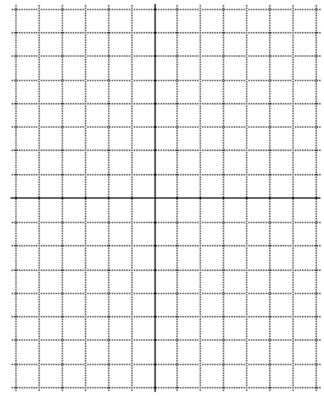
CHECK YOUR UNDERSTANDING

1. Solve this linear system.

$2x + 3y = 3$

$x - y = 4$

[Answer: (3, –1)]



Example 2: Solving a Problem by Graphing a Linear System

One plane left Regina at noon to travel 1400 mi. to Ottawa at an average speed of 400 mph. Another plane left Ottawa at the same time to travel to Regina at an average speed of 350 mph. A linear system that models this situation is:

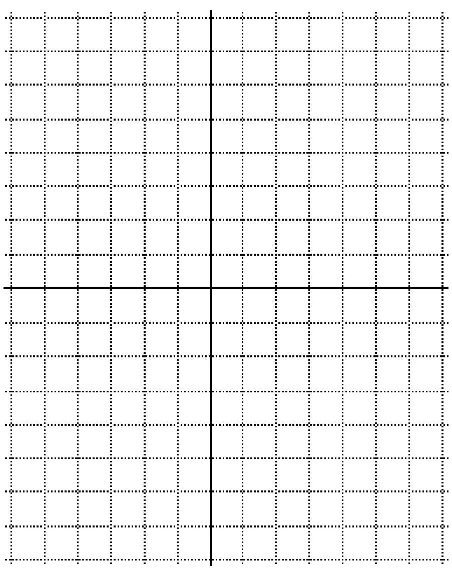
$d = 1400 - 400t$

$d = 350t$

where d is the distance in miles from Ottawa and t is the time in hours since the planes took off

a) Graph the linear system above.

b) Use the graph to solve this problem: When do the planes pass each other and how far are they from Ottawa?



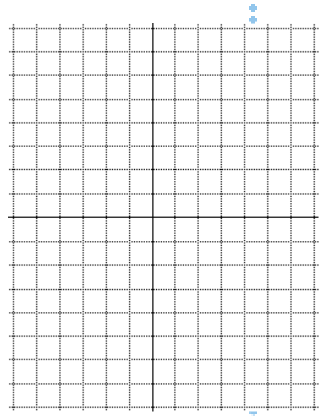
CHECK YOUR UNDERSTANDING

2. Jaden left her cabin on Waskesiu Lake, in Saskatchewan, and paddled her kayak toward her friend Tyrell's cabin at an average speed of 4 km/h. Tyrell started at his cabin at the same time and paddled at an average speed of 2.4 km/h toward Jaden's cabin. The cabins are 6 km apart. A linear system that models this situation is:

$d = 6 - 4t$

$d = 2.4t$

where d is the distance in kilometres from Tyrell's cabin and t is the time in hours since both people began their journey



a) Graph the linear system above.

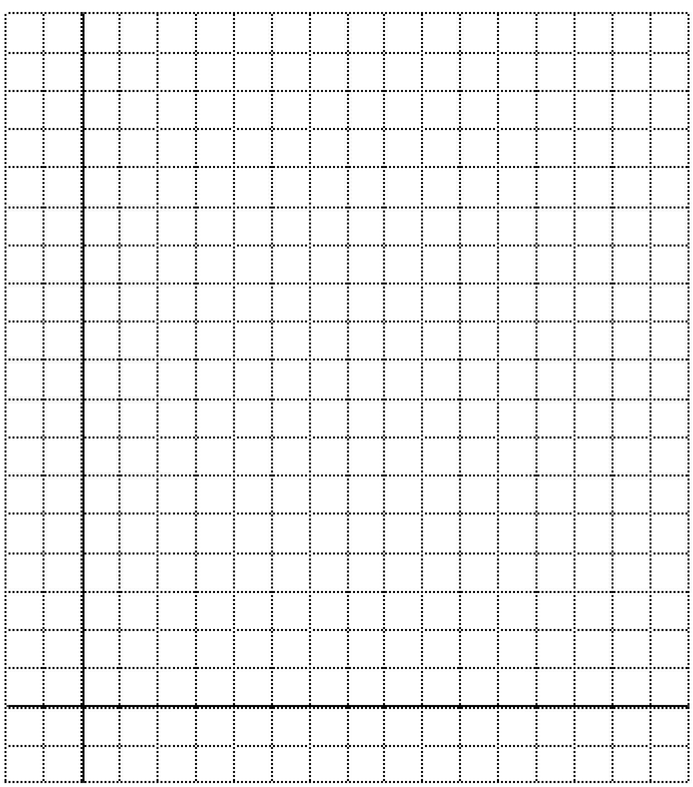
b) Use the graph to solve this problem: When do Jaden and Tyrell meet and how far are they from Tyrell's cabin?

[Answer: b) after travelling for approximately 54 min and at approximately 2.3 km from Tyrell's cabin]

Example 3: Solving a Problem by Writing then Graphing a Linear System

a) Write a linear system to model this situation:
To visit the Head-Smashed-In Buffalo Jump interpretive centre near Fort Macleod, Alberta, the admission fee is $5 for a student and $9 for an adult. In one hour, 32 people entered the centre and a total of $180 in admission fees was collected.

b) Graph the linear system then solve this problem: How many students and how many adults visited the centre during this time?

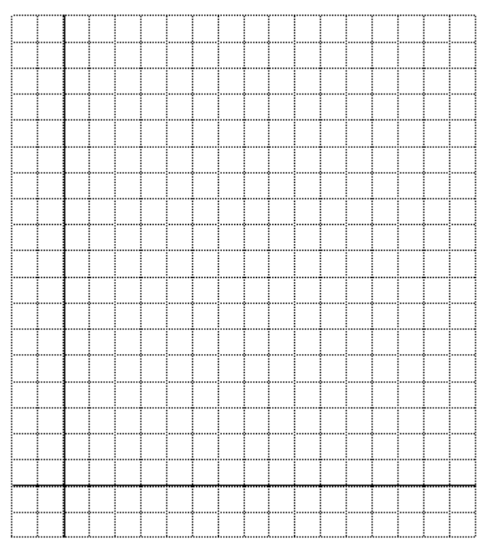


CHECK YOUR UNDERSTANDING

3. a) Write a linear system to model this situation: Wayne received and sent 60 text messages on his cell phone in one weekend. He sent 10 more messages than he received.

b) Graph the linear system then solve this problem: How many text messages did Wayne send and how many did he receive?

[Answers: a) $s + r = 60$; $s - r = 10$
b) 35 sent and 25 received]



Homework: Page409 #3, 4, 7–11, 13, 15, 16

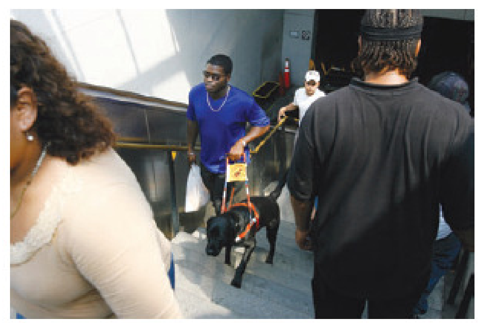
7.3 Math Lab: Using Graphing Technology to Solve a System of Linear Equations

LESSON FOCUS

Determine and verify the solution of a linear system using graphing technology.

Make Connections

In 2006, the population of Canada was 31 612 897. The population of the eastern provinces was 12 369 487 more than the population of the territories and western provinces.



What linear system models this situation?
How could you determine the population of the territories and western provinces?
How could you determine the population of the eastern provinces?
Why can't you determine an exact solution by graphing on grid paper?

Construct Understanding

TRY THIS

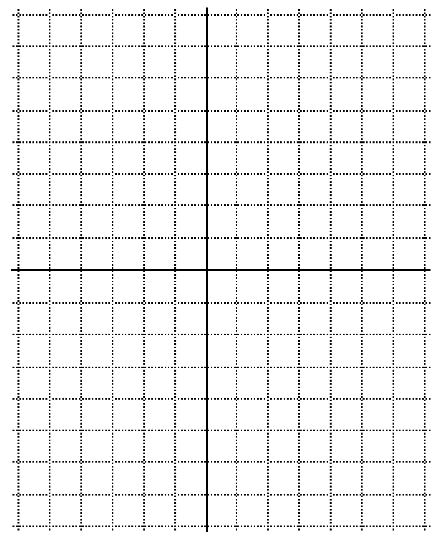
Work with a partner.
You will need:

- a graphing calculator or computer with graphing software

Léa's school had a carnival to celebrate *Festival du Voyageur*.
The school raised $1518.75 by charging an adult $3.75 and a student $2.50.
The total attendance was 520.
How many adults and how many students attended?

A. Write a linear system to model this situation.

B. Express each equation in slope-intercept form. Graph each line.



C. Determine the coordinates of the point of intersection of the lines. Are these coordinates exact or approximate? Explain.

D. How could you verify the solution in Step C by using tables of values?

E. Verify your solution by using the data in the given problem.

F. How can you use your results to determine the number of adults and the number of students who attended the carnival?

Homework: Page 415 #1–6

CHECKPOINT 1

Connections

Given this Situation and Related Problem

A store has boxes containing 1500 golf balls.

There are 5 more boxes containing 12 balls than boxes containing 24 balls.

How many of each size of box does the store have?

Identify the Variables

Let x represent the number of boxes with 12 balls.

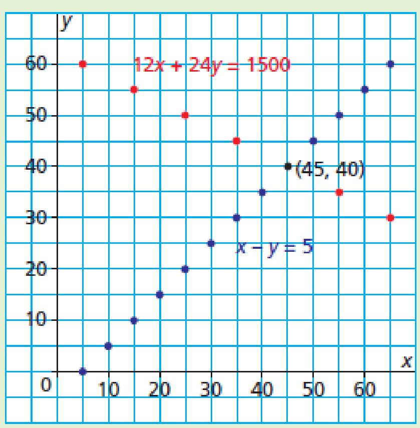
Let y represent the number of boxes with 24 balls.

Model the Situation with a Linear System

$12x + 24y = 1500$

$x - y = 5$

Graph to Locate the Point of Intersection



Identify the Solution

$x = 45$

$y = 40$

Use the equations to verify the solution.

Use the problem to verify the solution.

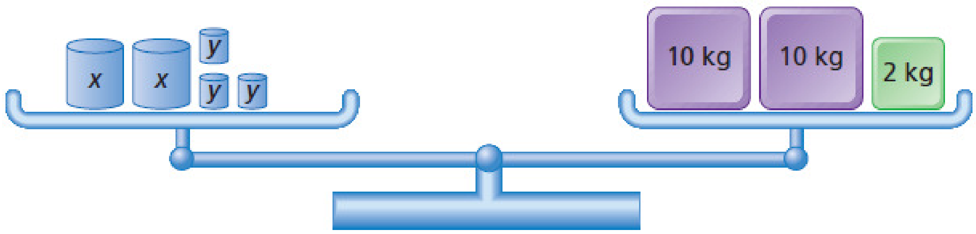
Concept Development

- **In Lesson 7.1**
  - You defined a linear system, wrote linear systems to model problems, and related linear systems to problems.
- **In Lesson 7.2**
  - You solved a linear system by graphing the linear equations on grid paper and determining the coordinates of their point of intersection.
  - You identified whether a solution to a linear system was exact or approximate.
- **In Lesson 7.3**
  - You solved linear systems using graphing technology by first expressing each equation in the form $y = f(x)$.
  - You used technology to determine whether the solution of a linear system was exact or approximate.
  - You verified the solution of a linear system by examining tables of values.

7.4 Using a Substitution Strategy to Solve a System of Linear Equations

LESSON FOCUS

Use the substitution of one variable to solve a linear system.



Make Connections

Look at the picture above.
Is there enough information to determine the masses of the container labelled x and the container labelled y? Explain.
How would your answer change if you knew that each container labelled x had a mass of 5 kg? Explain.

Construct Understanding

THINK ABOUT IT

Work with a partner.

Solve each linear system without graphing.

- $3x + 5y = 6$
  $x = -4$
- $2x + y = 5$
  $y = -x + 3$

What strategies did you use?
How can you check that each solution is correct?

Example 1: Solving a Linear System by Substitution

Solve this linear system.

$2x - 4y = 7$

$4x + y = 5$

CHECK YOUR UNDERSTANDING

1. Solve this linear system.

$5x - 3y = 18$

$4x - 6y = 18$

[Answer: $x = 3$ and $y = -1$]

Example 2: Using a Linear System to Solve a Problem

a) Create a linear system to model this situation:

Nuri invested $2000, part at an annual interest rate of 8% and the rest at an annual interest rate of 10%. After one year, the total interest was $190.

b) Solve this problem: How much money did Nuri invest at each rate?

CHECK YOUR UNDERSTANDING

2. a) Create a linear system to model this situation:

Alexia invested $1800, part at an annual interest rate of 3.5% and the rest at an annual interest rate of 4.5%. After one year, the total interest was $73.

b) Solve this problem: How much money did Alexia invest at each rate?

[Answers: a) $x + y = 1800$; $0.035x + 0.045y = 73$
b) Alexia invested $800 at 3.5% and $1000 at 4.5%.]

Example 3: Solving a Linear System with Fractional Coefficients

Solve this linear system by substitution.

$$\frac{1}{2}x + \frac{2}{3}y = -1$$

$$y = \frac{1}{4}x - \frac{5}{3}$$

CHECK YOUR UNDERSTANDING

3. Solve this linear system by substitution.

$$\frac{1}{2}x - \frac{4}{5}y = -2$$

$$y = \frac{1}{4}x - \frac{3}{8}$$

[Answer: $x = -\frac{23}{3}$ and $y = -\frac{55}{24}$]

Homework: Page 425 #4, 5, 7–9, 12, 13, 15, 16, 18–20

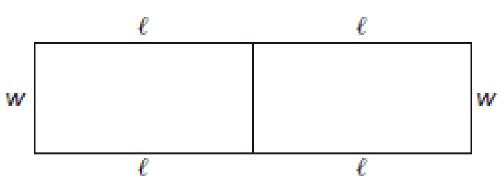
7.5 Using an Elimination Strategy to Solve a System of Linear Equations

LESSON FOCUS

Use the elimination of one variable to solve a linear system.

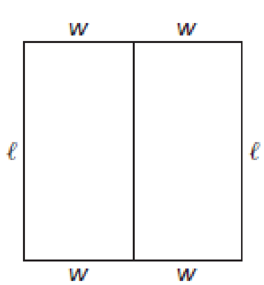
Make Connections

A carpenter placed two identical plywood sheets end to end and measured their perimeter.



Perimeter: 40 ft.

The carpenter placed the sheets side to side and measured their perimeter.



Perimeter: 32 ft.

Suppose you wanted to determine the dimensions of a piece of this plywood.
Which linear system would model the situation?
How would you solve the linear system?

Example 1: Solving a Linear System by Subtracting to Eliminate a Variable

Solve this linear system by elimination.

$3x - 4y = 7$
$5x - 6y = 8$

CHECK YOUR UNDERSTANDING

1. Solve this linear system by elimination.
   $2x + 7y = 24$
   $3x - 2y = -4$
   [Answer: $x = 0.8$ and $y = 3.2$]

Example 2: Solving a Linear System by Adding to Eliminate a Variable

Use an elimination strategy to solve this linear system.

$\frac{2}{3}x - \frac{1}{2}y = 4$
$\frac{1}{2}x + \frac{1}{4}y = \frac{5}{2}$

CHECK YOUR UNDERSTANDING

2. Use an elimination strategy to solve this linear system.
   $\frac{3}{4}x - y = 2$
   $\frac{1}{8}x + \frac{1}{4}y = 2$
   [Answer: $x = 8$ and $y = 4$]

Example 3: Using a Linear System to Solve a Problem

a) Write a linear system to model this situation:
An alloy is a mixture of metals. An artist was commissioned to make a 100-g bracelet with a 50% silver alloy. He has a 60% silver alloy and a 35% silver alloy.

b) Solve this problem:
What is the mass of each alloy needed to produce the desired alloy?

CHECK YOUR UNDERSTANDING

3. a) Write a linear system to model this situation:
An artist was commissioned to make a 625-g statue of a raven with a 40% silver alloy. She has a 50% silver alloy and a 25% silver alloy.

b) Solve this problem: What is the mass of each alloy needed to produce the desired alloy?

[Answers: a) $f + t = 625$; $0.50f + 0.25t = 250$
b) 375 g of the 50% alloy; 250 g of the 25% alloy]

Example 4: Solving by Determining the Value of Each Variable Independently

Solve this linear system.
$2x + 3y = 8$
$5x - 4y = -6$

CHECK YOUR UNDERSTANDING

4. Solve this linear system.
$3x + 9y = 5$
$9x - 6y = -7$
[Answer: $x = -\frac{1}{3}$ and $y = \frac{2}{3}$]

Homework: Page 437 #6–10, 12, 17

CHECKPOINT 2

Connections

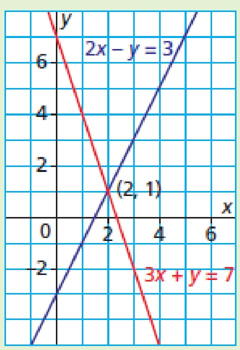
Linear System

$3x + y = 7$

$2x - y = 3$

Solve by graphing

Use graphing software, a graphing calculator, or grid paper.



Solve by substitution

$3x + y = 7$ ①

$2x - y = 3$ ②

Solve equation ① for y.

$y = -3x + 7$

Substitute for y in equation ②.

$2x - (-3x + 7) = 3$

$5x - 7 = 3$

$5x = 10$

$x = 2$

Substitute for x in equation ①.

$3(2) + y = 7$

$6 + y = 7$

$y = 1$

Solution: $x = 2$ and $y = 1$

Solve by elimination

$3x + y = 7$ ①

$2x - y = 3$ ②

Add the equations to eliminate y.

$5x = 10$

$x = 2$

Substitute for x in equation ①.

$3(2) + y = 7$

$6 + y = 7$

$y = 1$

Solution: $x = 2$ and $y = 1$

Verify the solution.

Substitute for x and y in each equation to check that the values satisfy the equations.

$3x + y = 7$ ①	$2x - y = 3$ ②
L.S. $= 3x + y$	L.S. $= 2x - y$
$= 3(2) + 1$	$= 2(2) - 1$
$= 7$	$= 3$
$=$ R.S.	$=$ R.S.

Since the left side is equal to the right side for each equation, the solution is: $x = 2$ and $y = 1$

Concept Development

- **In Lesson 7.4**
  - You solved linear systems by using the substitution strategy.
  - You created equivalent equations by multiplying or dividing each term in an equation by a non-zero number.
- **In Lesson 7.5**
  - You solved linear systems by using the elimination strategy.
  - You created an equivalent linear system by adding or subtracting the two equations in a linear system.

7.6 Properties of Systems of Linear Equations

LESSON FOCUS

Determine the numbers of solutions of different types of linear systems.

Make Connections

Phil was teased by his grandparents to determine their ages given these clues.

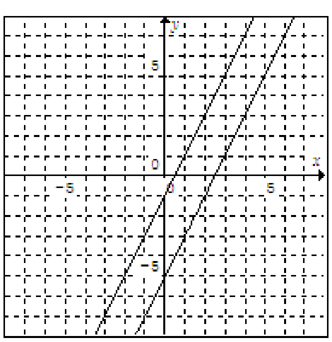
The sum of our ages is 151.

Add our two ages. Double this sum is 302.

What are our ages?

Can Phil determine his grandparents' ages given these clues? Why or why not?

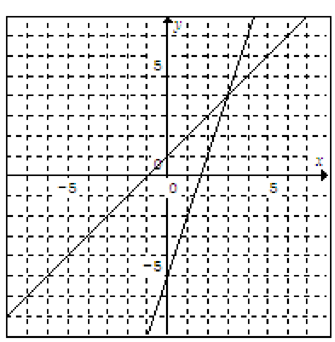
Parallel lines
Inconsistent



$y = 2x - 1$

$y = 2x - 5$

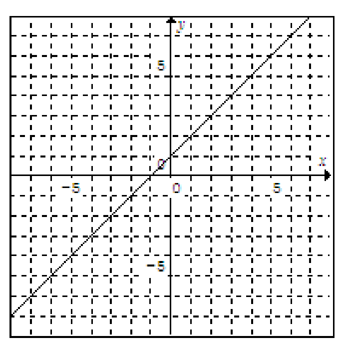
Intersecting Lines
Consistent



$y = x + 1$

$y = 3x - 5$

Coincident Lines
Consistent



$x - y + 1 = 0$

$3x - 3y + 3 = 0$

Example 1: Determining the Number of Solutions of a Linear System

Determine the number of solutions of each linear system.

a) $x + y = -2$
$-2x - 2y = 4$

b) $4x + 6y = -10$
$-2x - y = -1$

c) $3x + y = -1$
$-6x - 2y = 12$

CHECK YOUR UNDERSTANDING

1. Determine the number of solutions of each linear system.

a) $x + y = 3$
$-2x - y = -2$

b) $4x + 6y = -10$
$-2x - 3y = 5$

c) $2x - 4y = -1$
$3x - 6y = 2$

[Answers: a) one solution
b) infinite solutions
c) no solution]

Example 2: Creating a Linear System with 0, 1, or Infinite Solutions

Given the equation $-2x + y = 4$, write another linear equation that will form a linear system with:

a) exactly one solution

b) no solution

c) infinite solutions

CHECK YOUR UNDERSTANDING

2. Given the equation $-6x + y = 3$, write another linear equation that will form a linear system with:

a) exactly one solution

b) no solution

c) infinite solutions

[Sample Answers: a) $y = 2x + 4$
b) $y = 6x - 3$ c) $-12x + 2y = 6$]

Homework: Page 448 #6, 7, 11, 15, 17

STUDY GUIDE

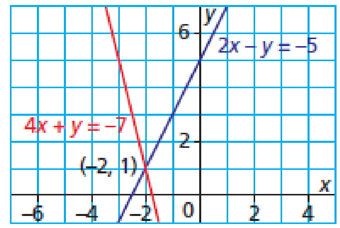
CONCEPT SUMMARY

Big Ideas	Applying the Big Ideas
■ A system of two linear equations is solved when the set of ordered pairs that satisfies both equations is determined.	This means that: ■ Substituting (x, y) into the two equations of a linear system determines whether the ordered pair is a solution. You can determine this solution using graphs, or algebraic strategies.
■ Multiplying or dividing the equations in a linear system by a non-zero number, or adding or subtracting the equations produces an equivalent system.	■ You can create an equivalent system by multiplying or dividing each term in a linear equation by a constant. The solution of the new system is the same as the original solution. You can create an equivalent system by adding or subtracting the like terms in the equations of a linear system. The solution of the new system is the same as the original system.
■ A system of two linear equations may have one solution, infinite solutions, or no solution.	■ You can determine the number of solutions by graphing or by comparing the slopes and y-intercepts of the graphs of the linear equations.

Reflect on the Chapter

- How do you determine the number of solutions of a linear system?
- What are some different strategies you can use to solve a linear system?
- Why does multiplying or dividing each term in each equation or adding or subtracting the equations not change the solution of a linear system?

SKILLS SUMMARY

Skill	Description	Example
Solve a linear system by graphing. [7.2, 7.3]	To solve a linear system by graphing: **1.** Draw the graphs after determining their intercepts, or their slopes and y-intercepts. **2.** The coordinates of the point of intersection are the solution of the linear system. **3.** Verify the solution by substituting the coordinates into the equations.	For this linear system: $2x - y = -5$ $4x + y = -7$  The solution is: $x = -2$ and $y = 1$
Solve a linear system algebraically. [7.4, 7.5]	To solve a linear system algebraically: **1.** Use substitution or elimination. **2.** Verify the solution by substituting for x and y in both equations to check that the coordinates of the point of intersection satisfy both equations.	For this linear system: $2x - y = -5$ ① $4x + y = -7$ ② Use elimination. Add the equations. $6x = -12$ $x = -2$ Substitute for x in equation ①. $2(-2) - y = -5$ $y = 1$ The solution is: $x = -2$ and $y = 1$
Determine the number of solutions of a linear system. [7.6]	To determine the number of solutions in a linear system, use Step 1 or Step 2: **1.** Compare the graphs of the equations. **2.** Compare the slopes and y-intercepts of the lines.	The graphs of the linear system above have different slopes, so there is exactly one solution. The graphs of the linear system below have the same slope and the same y-intercept, so there are infinite solutions: $2x + 4y = 6$ $4x + 8y = 12$ The graphs of the linear system below have the same slope and different y-intercepts, so there is no solution: $2x + 4y = 6$ $4x + 8y = 10$

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