6.1 – Solving Equations by Using Inverse Operations

Focus: Model a linear equation problem and solve pictorially and algebraically.

Recall: State the inverse operation of the following.

Addition → ____________________ Subtraction → ____________________

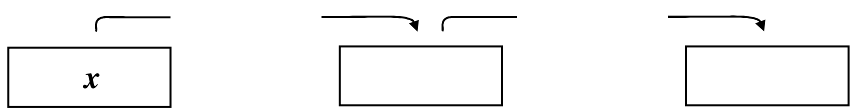
Multiplication → ____________________ Division → ____________________

When we solve an equation, we are looking for the value of the variable that will satisfy the equation.

In other words we are looking for the value of the variable, when substituted into the equation will make the left side equal to the right side.

Let us examine the equation $3x+2=11$.

To build the equation, let's begin with the x variable

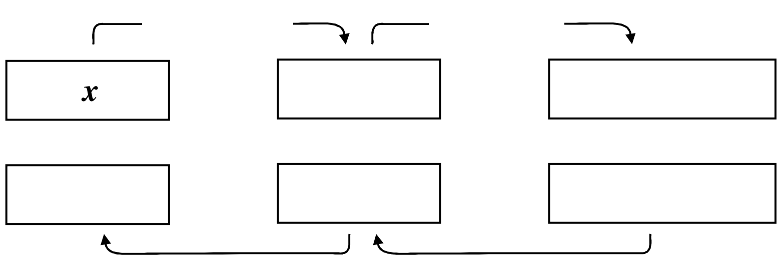


We are looking for the value of $\boldsymbol{x}$ that when substituted into the equation will make the left side of the equation equal to 11.

Solving Equations

To solve an equation, we apply the inverse operations to the equation in the reverse order.

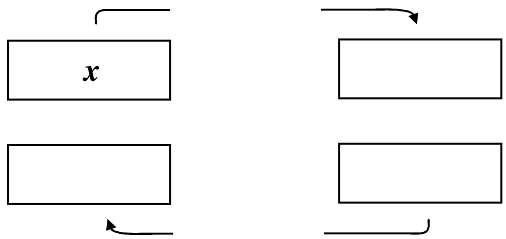
Ex. 1: Solve the equation $3x + 2 = 11$



Solve Algebraically:

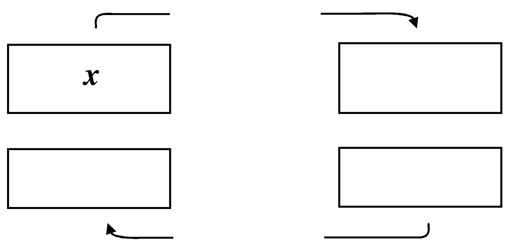
$$3x + 2 = 11$$

Ex. 2: Solve the equation $-3.4x = 23.8$



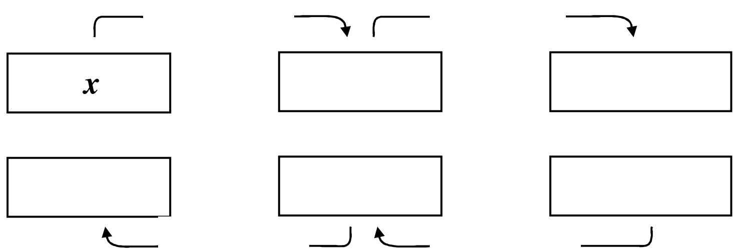
Algebraically: $-3.4x = 23.8$

Ex. 3: Solve the equation $\frac{x}{5} = -2.1$



Algebraically: $\frac{x}{5} = -2.1$

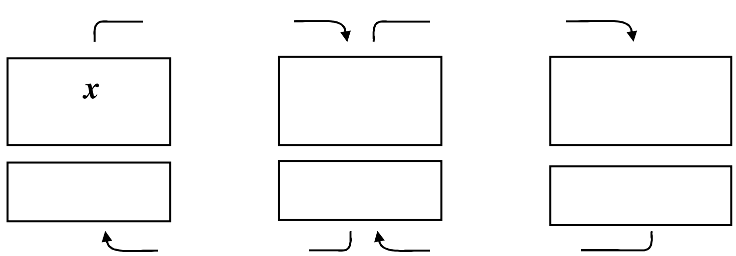
Ex. 4: Solve the equation $5.2x + 10.8 = -7.4$



Algebraically:

$5.2x + 10.8 = -7.4$

Ex. 5: Solve the equation $\frac{x}{2} + 2.1 = 9.5$



Algebraically:

$\frac{x}{2} + 2.1 = 9.5$

Ex. 6: Solve the equation $-2(x+4)=12$ in two different ways and verify your solution.

I. $-2(x + 4) = 12.6$

II. $-2(x + 4) = 12.6$

Ex. 7:
If you triple a number, then add fifteen, the result is negative twelve.
What is the number?

a) If n represents the number, write the equation.

b) Solve the equation.

c) Verify your solution.

Ex. 8:
12% of a number is 5.4.
What is the number?

a) If c represents the number, write the equation.

b) Solve the equation.

c) Verify your solution.

HW Assignment
Section 6.1 pg. 271 # 5bcd, 6, 7, 8def, 9, 10bdf, 11, 13, 15, 17, 18, 20, 21

6.2 – Solving Equations by Using Balance Strategies

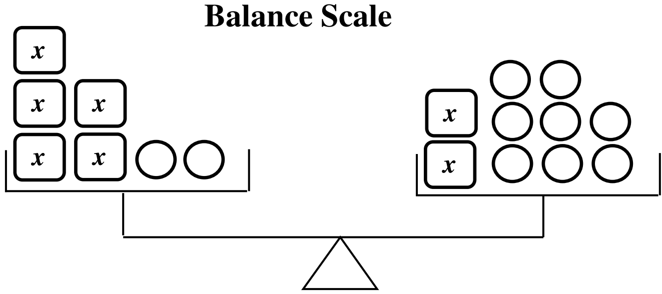
Focus: Model a linear equation problem and solve pictorially and algebraically.

Last day we isolated the variable by performing the inverse operations in the reverse order.

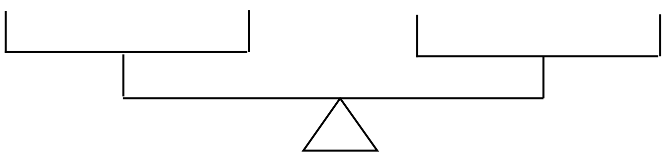
However, this strategy can only be used when the variable occurs once in the equation. When variables occur on both sides of the equation we will use balance scales and algebra tiles.

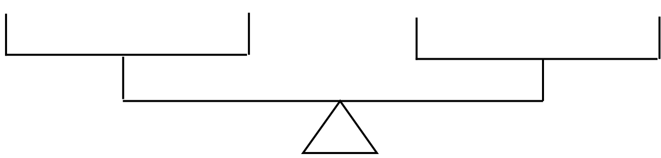
Let us examine the equation $5x + 2 = 2x + 8$.
We will model the solution using the scale as well as present an algebraic solution.

Balance Scale **Algebraic Solution**

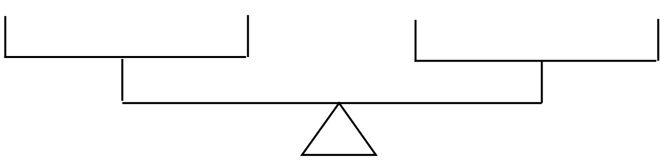


$5x + 2 = 2x + 8$

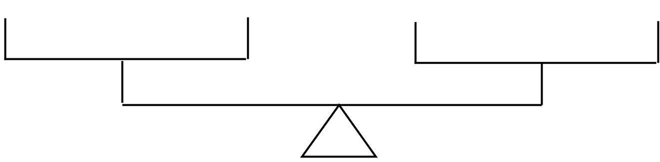


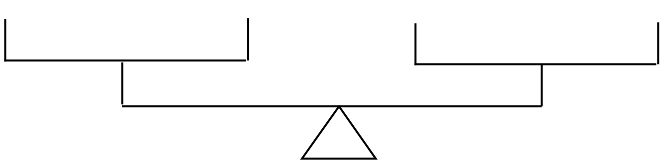


Ex. 1: Model the equation $8x + 3 = 4x + 7$ using a scale and algebraically.



$8x + 3 = 4x + 7$

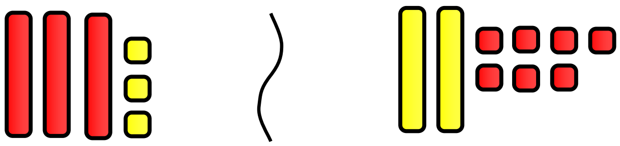




The scale model works for algebraic equations where all the terms are positive. To model an equation that contain negative terms we will use algebra tiles.

Ex. 2: Solve the equation $-3x + 3 = 2x - 7$ using algebra tiles

Algebra Tiles

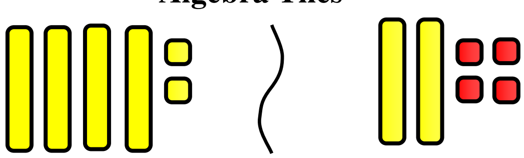


Algebraic Solution

$-3x + 3 = 2x - 7$

Ex. 3: Solve the equation $4x + 2 = 2x - 4$ using algebra tiles.

Algebra Tiles



Algebraic Solution

$4x + 2 = 2x - 4$

Equations that contain fractions or decimals cannot be easily modeled with balance scales or algebra tiles. We can solve these equations by doing the same operation to both sides of the equation to isolate the variable.

To solve an equation, we need to isolate the variable, meaning get the variable on one side of the equation. To do this we can perform the following operations:

- ____________ the same quantity to each side.
- ______________________ the same quantity from each side.
- ___________________ or ____________________each side by the same non-zero quantity.

If an equation has any terms in fraction form, multiply both sides of the equation by the lowest common denominator (LCD) to create an equivalent equation without fractions. Then solve as usual.

Ex. 4: Solve and verify your solution.

$$8.8x + 2.1 = 2.3x - 16.1$$

Ex. 5: Solve and verify your solution.

$$8(e+1) = 6(e+3)$$

Ex. 6: Solve and verify your solution.

$$-6(c-2) = 2(c+18)$$

Ex. 7: Solve and verify your solution.

$$\frac{136}{x} = -17 \quad , \quad x \neq 0$$

Ex. 8: Solve the equation.

$$\frac{2}{3}(x+4) = -4(x-1)$$

Ex. 9: Solve the equation.

$$\frac{x}{5} + \frac{1}{2} = \frac{3}{10}$$

Ex. 10: Solve the equation.

$$\frac{4x}{5} + 7 = \frac{2x}{3}$$

Ex. 11: Ben and Hines want to rent scooters while on a vacation. They come across two rental shops with the following rates:

Scooter-World $17 for the first hour, $16 for each additional hour.
Vespa-Ville $35 for the first hour, $12 for each additional hour

Ben decides to rent from Scooter-World and Hines rents from Vespa-Ville. How long would they have to ride for to pay the exact same rental amount?

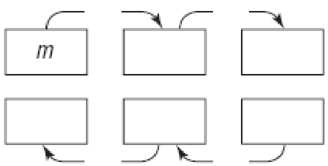
HW Assignment
Section 6.2 pg. 280 # 4 – 6, 8, 9, 10ace, 11ace, 13, 14, 17 – 21
Quiz next class on 6.1 & 6.2

6.1

1. For each equation, write the first operation you would use to isolate the variable. Justify your choice of operation.
   a) $-3j = 9.6$ b) $\frac{1}{4}r - 2 = 4$
   c) $2(-3x + 1.5) = 6$ d) $3 = -2n + 9$

2. Marshall creates this arrow diagram to show the steps in the solution of $\frac{m}{10} + 20.3 = 45.5$.

Build equation

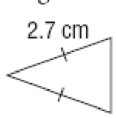


Solve equation

   a) Copy and complete the arrow diagram.
   b) Record the solution algebraically.

3. Sheila is charged a fare of \$27.70 for a cab ride to her friend's house. The fare is calculated using a flat fee of \$2.50, plus \$1.20 per kilometre. What distance did Sheila travel?
   a) Let k kilometres represent the distance travelled. Write an equation to solve the problem. Solve the problem.
   b) Verify the solution.

4. An isosceles triangle has two equal sides of length 2.7 cm and perimeter 7.3 cm.

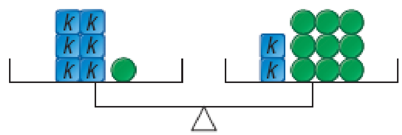


   a) Write an equation that can be used to determine the length of the third side.
   b) Solve the equation.
   c) Verify the solution.

5. Solve each equation. Verify the solution.
   a) $\frac{k}{3} = -1.5$
   b) $10.5 = 3b - 12.5$
   c) $5(x - 7.2) = 14.5$
   d) $8.4 = 1.2b$
   e) $2 + \frac{n}{3} = 2.8$
   f) $-8 = 0.4(3.2 + h)$

6.2

6. Write the equation modelled by these balance scales. Solve the equation. Verify the solution.



Each ● has a mass of 1 g.

7. Solve each equation. Verify the solution.
   a) $\frac{56}{a} = -3.5, a \neq 0$
   b) $8w - 12.8 = 6w$
   c) $-8z + 11 = -10 - 5.5z$
   d) $\frac{5x}{2} = 11 + \frac{2x}{3}$
   e) $0.2(5 - 2r) = 0.3(1 - r)$
   f) $12.9 + 2.3y = 4.5y + 19.5$
   g) $\frac{2}{5}(m + 4) = \frac{1}{5}(3m + 9)$

8. Skateboards can be rented from two shops in a park.

   Shop Y charges \$15 plus \$3 per hour
   Shop Z charges \$12 plus \$4 per hour

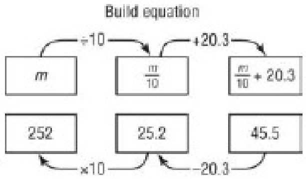
   Determine the time in hours for which the rental charges in both shops are equal.
   a) Write an equation to determine the time.
   b) Solve the equation.
   c) Verify the solution.

Answers:

1. a) Divide by −3. b) Add 2.
   c) Divide by 2. d) Subtract 9.

2. a)

Build equation



Solve equation

   b)
$$\frac{m}{10} + 20.3 = 45.5$$
$$\frac{m}{10} + 20.3 - 20.3 = 45.5 - 20.3$$
$$\frac{m}{10} = 25.2$$
$$\frac{m}{10} \times 10 = 25.2 \times 10$$
$$m = 252$$

3. a) $2.5 + 1.2k = 27.7$; $k = 21$
   Sheila travelled 21 km.

4. a) Let s represent the length of the third side in centimetres: $2(2.7) + s = 7.3$, or $5.4 + s = 7.3$
   b) $s = 1.9$

5. a) $k = -4.5$ b) $b = 7\frac{2}{3}$
   c) $x = 10.1$ d) $b = 7$
   e) $n = 2.4$ f) $h = -23.2$

6. $6k + 1 = 2k + 9$; $k = 2$

7. a) $a = -16$ b) $w = 6.4$
   c) $z = 8.4$ d) $x = 6$
   e) $r = 7$ f) $y = -3$
   g) $m = -1$

8. a) Let t represent the time in hours. $15 + 3t = 12 + 4t$
   b) $t = 3$

6.3 – Introduction to Linear Inequalities

Focus: Write and graph linear inequalities.

Recall: What do each of the following signs mean?

a) $>$

b) $<$

c) $\geq$

d) $\leq$

Warm-up:

Define a variable and write an inequality for each situation.

a)

b)

c)

d)

Ex. 1: Define a variable and write an inequality to describe each situation:

a) Your bank account balance has been above $300 the whole year.

b) You must have 9 items or less to use the express checkout.

c) Contest entrants must be at least 13 years old.

d) Tiger Woods shot below –1 everyday of the tournament.

What is different about an inequality compared to an equation?

Ex. 2: Write how you would say the following inequalities, then give 2 possible solutions.

a) $x > 2$

b) $x \leq 7$

c) $x < -9$

d) $-3 \geq x$

Ex. 3: Is each number a solution of the inequality $y \geq -3$?

a) -4 b) 4 c) -2.5 d) 0 e) -3

How can you illustrate solutions to an inequality?

Graphing on a Number Line (with the variable on left side of inequality)

Set up the number line with the target number in the middle.

- If $>$ or $<$, use ○ on target number.
- If $\geq$ or $\leq$, use ● on target number.
- If $>$ or $\geq$, use right arrow $\longrightarrow$
- If $<$ or $\leq$, use left arrow $\longleftarrow$

Ex. 4: Graph each inequality on a number line and write two possible solutions.

a) $m < -1$

b) $0.5 < p$

c) $w \leq -3.5$

d) $t \geq 0$

HW Assignment
Section 6.3 pg. 292 # 3 – 16

6.4 – Solving Linear Inequalities by Using Addition and Subtraction

Focus: Use Addition & Subtraction to solve inequalities

Warm-up: Do the Investigate on of p. 294

Read 'Connect' on p.295 and give one summarizing sentence.

To solve an inequality, we use the same strategy as for solving an equation.
We will isolate the variable by adding to or subtracting from each side of the inequality.

Ex. 1: Solve and graph the following inequalities:

a) $x + 3 \geq 5$

b) $m - 2.3 < -1.2$

c) $4.1 \leq p - 3.1$

Ex. 2: Do a check for Ex. 1a

Why aren't checks perfect for inequalities?

Ex. 3: Solve and graph the inequalities:

a) $5x - 3 < 4x + 7$

b) $-9y + 1 > -10y + 1$

Ex. 4: Rufus plans to board his dog while he is away on vacation. Puppy-Palace charges \$90 plus \$5 per day. Doggy-Day-Care charges \$100 plus \$4 per day. For how many days can Rufus board his dog so that Puppy-Palace is less expensive than Doggy-Day-Care?

a) Choose a variable and write an expression for the cost of boarding at each establishment.

b) Write an inequality.

c) Solve the problem.

d) Graph the solution.

HW Assignment
Section 6.4 pg. 298 # 4, 6 – 9, 11 – 14

6.5 – Solving Linear Inequalities by Using Multiplication and Division

Focus: Use Multiplication & Division to solve inequalities

Investigate:

In the patterns below, each side of the inequality $12 > 6$ is multiplied or divided by the same non-zero number.

Multiplication Pattern	Division Pattern
$12 > 6$	$12 > 6$
$12(-3) \square 6(-3)$	$12 \div (-3) \square 6 \div (-3)$
$12(-2) \square 6(-2)$	$12 \div (-2) \square 6 \div (-2)$
$12(-1) \square 6(-1)$	$12 \div (-1) \square 6 \div (-1)$
$12(1) \square 6(1)$	$12 \div 1 \square 6 \div 1$
$12(2) \square 6(2)$	$12 \div 2 \square 6 \div 2$
$12(3) \square 6(3)$	$12 \div 3 \square 6 \div 3$

- simplify each expression in the patterns.
- Replace each $\square$ with $<$ or $>$ to create a true statement.
- Compare the inequality signs in the pattern with the inequality sign in $12 > 6$.
  When did the inequality sign stay the same?
  When did the inequality sign change?

What two important properties about inequalities resulted from the Investigate?

1.

2.

To solve an inequality, we use the same strategy as for solving an equation.
The only difference is when we multiply or divide by a negative number, we must reverse the inequality sign

Ex. 1: Solve and graph the following inequalities:

a) $3x \leq 18$ b) $-m < 4$ c) $\frac{t}{-5} \leq -1$ d) $\frac{w}{6} < 0$

Ex. 2: Solve and graph the inequality. Verify the solution.

a) $15 + 4x \geq 10x - 15$

b) $-2.6a + 14.6 < -5.2 + 1.8a$

Ex. 3: Solve and graph.

$$\frac{x}{4} + \frac{7}{4} > \frac{5}{6}$$

Ex. 4: Ice-World Ski Resort charges $4.50 to rent an inner tube and $0.65 each time you use the lift to get back to the top of the mountain. If Hakeem has $14.25, how many rides can he go on?

a) Choose a variable and write an inequality

b) Solve the problem

c) Graph the solution

HW Assignment
Section 6.5 pg. 305 # 3, 7 – 13, 15, 16

Study Guide

Solving Equations

- An equation is a statement that one quantity is equal to another. To solve an equation means to determine the value of the variable that makes the right side of the equation equal to the left side.

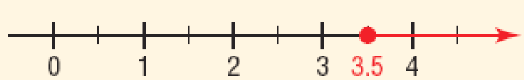
- To solve an equation, isolate the variable on one side of the equation. We use inverse operations or a balance strategy of performing the same operation on both sides of the equation. This can include:
  - adding the same quantity to each side of the equation
  - subtracting the same quantity from each side of the equation
  - multiplying or dividing each side of the equation by the same non-zero quantity

- Algebra tiles, arrow diagrams, and balance scales help model the steps in the solution.

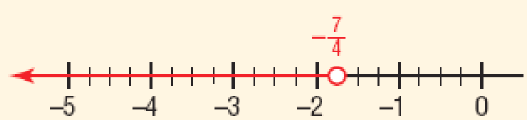
Solving Inequalities

- An inequality is a statement that:
  - one quantity is less than another; for example, $-4 < 3.2a$
  - one quantity is greater than another; for example, $\frac{3}{2}b + 8 > -7$
  - one quantity is greater than or equal to another; for example, $3.4 - 2.8c \geq 1.3c$
  - one quantity is less than or equal to another; for example, $-\frac{5}{8}d + \frac{1}{4} \leq \frac{3}{4} - \frac{1}{2}d$

- The solutions of an inequality are the values of the variable that make the inequality true. We can graph the solutions of an inequality on a number line; for example, $f \geq 3.5$:



  and $g < -\frac{7}{4}$:



- The inequality sign reverses when you multiply or divide each side of the inequality by the same negative number.

HW Assignment
Review pg. 308 # 1 – 16