6.1 Slope of a Line

LESSON FOCUS:
Determine the slope of a
line segment and a line.

Make Connections

The town of Falher in Alberta is known as
la capitale du miel du Canada, the Honey
Capital of Canada. It has the 3-story slide
in the photo. How could you describe the
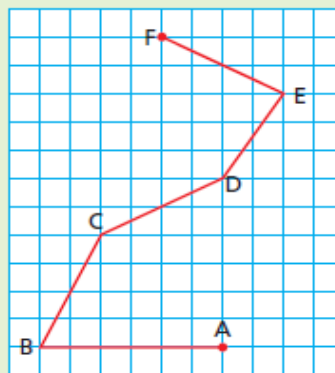steepness of the slide?

Construct Understanding

TRY THIS

Work with a partner.

This diagram shows different line segments
on a square grid.

A. Think of a strategy to calculate a number
to represent the steepness of each line
segment.

B. Which is the steepest line segment?
How does your number show that?

C. Which segment is the least steep? How does its number compare
with the other numbers?

D. On a grid, draw a line segment that is steeper than segment CD,
but not as steep as segment BC. Use your strategy to calculate
a number to represent its steepness.

E. How are line segments CD and EF alike and different?
How do the numbers for their steepnesses compare?

F. What number would you use to describe the steepness of
a horizontal line?

Some roofs are steeper than others. Steeper roofs are more expensive to shingle.
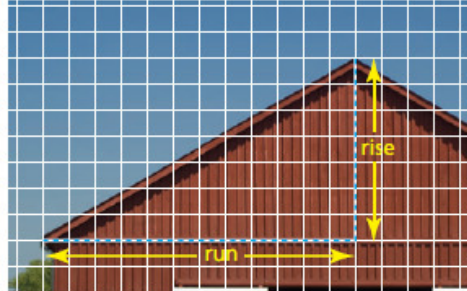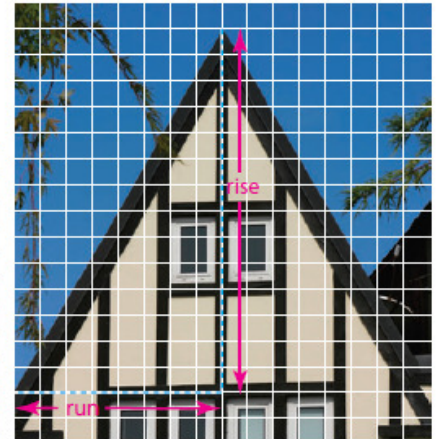
Roof A	Roof B	Roof C

The steepness of a roof is measured by calculating its **slope.**

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

The **rise** is the vertical distance from the bottom of the edge of the roof to the top.
The **run** is the corresponding horizontal distance.
For each roof above, we count units to determine the rise and the run.

For Roof A

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

For Roof B

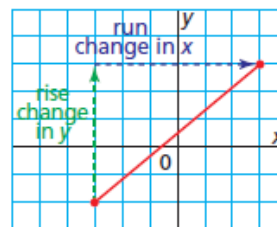$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

For Roof C

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

The slope of a line segment on a coordinate grid is the measure of its rate of change. From Chapter 5, recall that:

$$\text{Rate of change} = \frac{\text{change in dependent variable}}{\text{change in independent variable}}$$

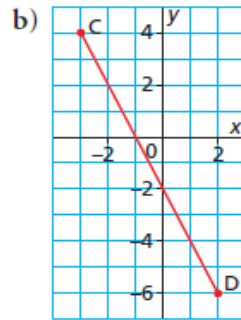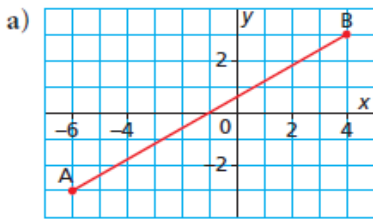$$\text{Rate of change} = \frac{\text{change in } y}{\text{change in } x}$$

The change in y is the rise.
The change in x is the run.
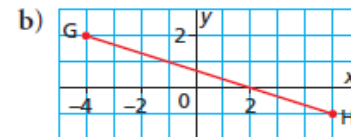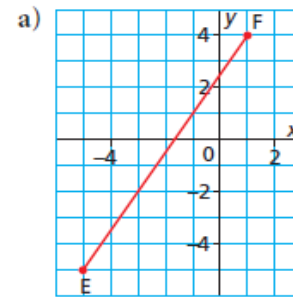So, slope $= \dfrac{\text{rise}}{\text{run}}$

Example 1: Determining the Slope of a Line Segment
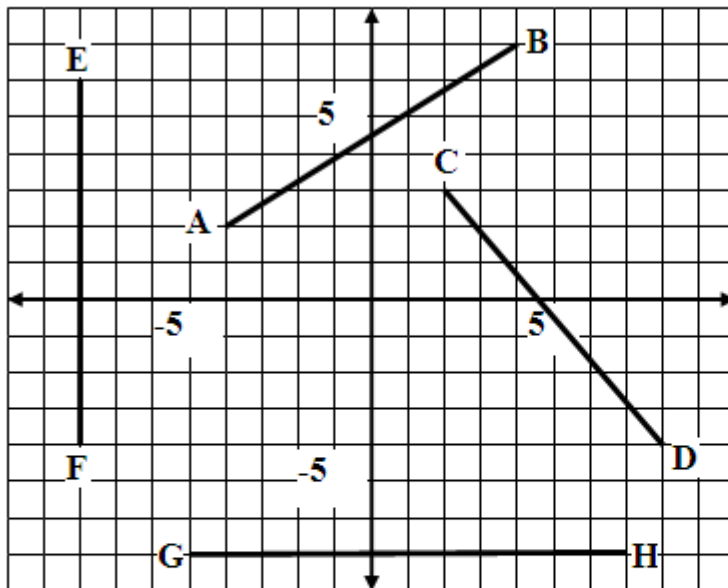
Determine the slope of each line segment.

a)

b)

CHECK YOUR UNDERSTANDING

1. Determine the slope of each line segment.

a)

b)

[Answers: a) $\frac{3}{2}$ b) $-\frac{1}{3}$]

Determine the slope of each line segment on the grid below.

- The slope of any horizontal line segment (like GH) is

  _____.

- The slope of any vertical line segment (like EF) is

  _____.

- A slope of a line segment (like AB) rising to the right is

  _____.

- A slope of a line segment (like CD) falling to the right is
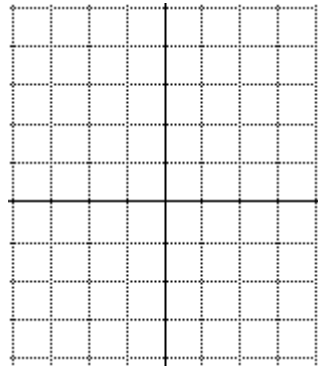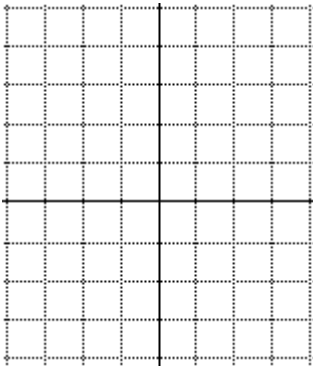
  _____.

AB   CD   EF   GH

3

Example 2: Drawing a Line Segment with a Given Slope

Draw a line segment with each given slope.
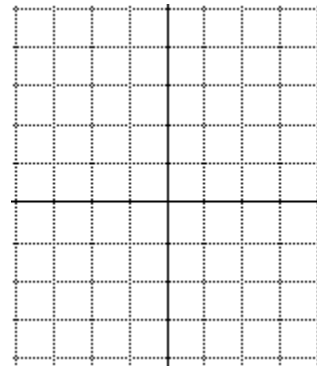
a) $\dfrac{7}{5}$
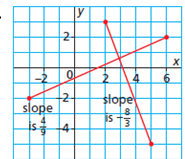
b) $-\dfrac{3}{8}$

CHECK YOUR UNDERSTANDING

2. Draw a line segment with each slope.

a) $\dfrac{4}{9}$

b) $-\dfrac{8}{3}$

Sample Answers:

slope is $\dfrac{4}{9}$ slope is $-\dfrac{8}{3}$

We can show that the slopes of all segments of a line are equal.
On line MT, vertical and horizontal segments are drawn for the rise and run.
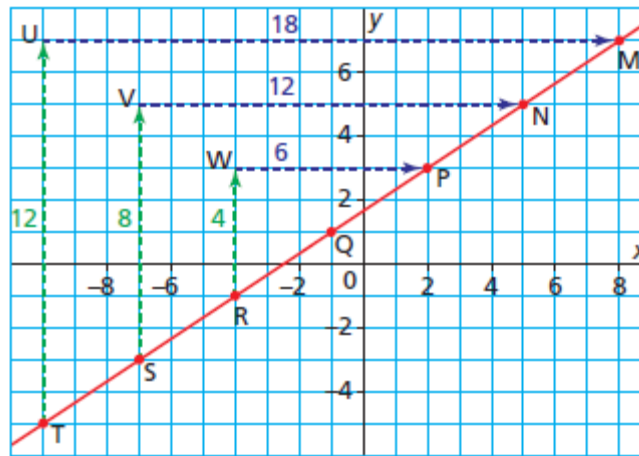These segments form right triangles.
Consider the lengths of the legs
of these right triangles.

$\dfrac{TU}{UM} = \dfrac{12}{18}$ $\dfrac{SV}{VN} = \dfrac{8}{12}$ $\dfrac{RW}{WP} = \dfrac{4}{6}$

$\dfrac{TU}{UM} = \dfrac{2}{3}$ $\dfrac{SV}{VN} = \dfrac{2}{3}$ $\dfrac{RW}{WP} = \dfrac{2}{3}$

The lengths of the legs have the
same ratio.
So, the triangles are similar.

Any right triangle drawn with its hypotenuse on line MT will have legs in the

ratio $\dfrac{2}{3}$. So it does not matter which points we choose on the line; the slope of

the line is the slope of any segment of the line. For example,
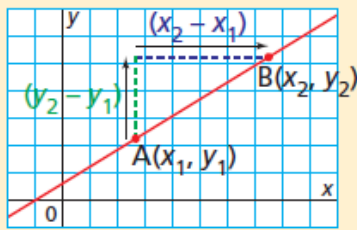
Slope of segment PQ $= \dfrac{2}{3}$ Slope of segment NR $= \dfrac{6}{9}$, or $\dfrac{2}{3}$

So, the slope of line MT is $\dfrac{2}{3}$.

4

Slope of a Line

A line passes through $A(x_1, y_1)$ and $B(x_2, y_2)$.

Slope of line AB $= \dfrac{y_2 - y_1}{x_2 - x_1}$

Example 3: Determining Slope Given Two Points on a Line

Determine the slope of the line that passes through $C(-5, -3)$ and $D(2, 1)$.

Example 4:

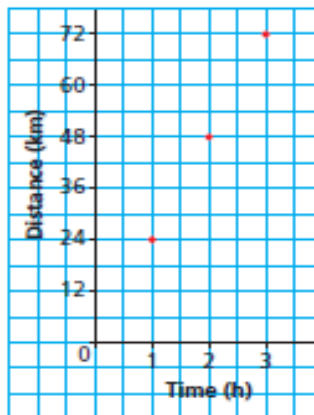Yvonne recorded the distances she had travelled at certain times since she began her cycling trip along the Trans Canada Trail in Manitoba, from North Winnipeg to Grand Beach. She plotted these data on a grid.

Graph of a Bicycle Ride
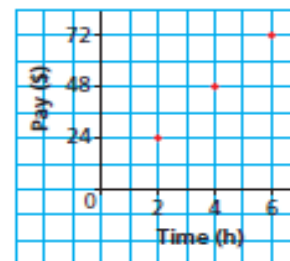
a) What is the slope of the line through these points?

b) What does the slope represent?

c) How can the answer to part b be used to determine:

   i) how far Yvonne travelled in $1\frac{3}{4}$ hours?

   ii) the time it took Yvonne to travel 55 km?

Homework: Page 339   #4–6, 8, 13, 15, 17–19, 22–24, 26, 28

6.2 Slopes of Parallel and Perpendicular Lines

LESSON FOCUS: Use slope to determine whether two lines are parallel or perpendicular.
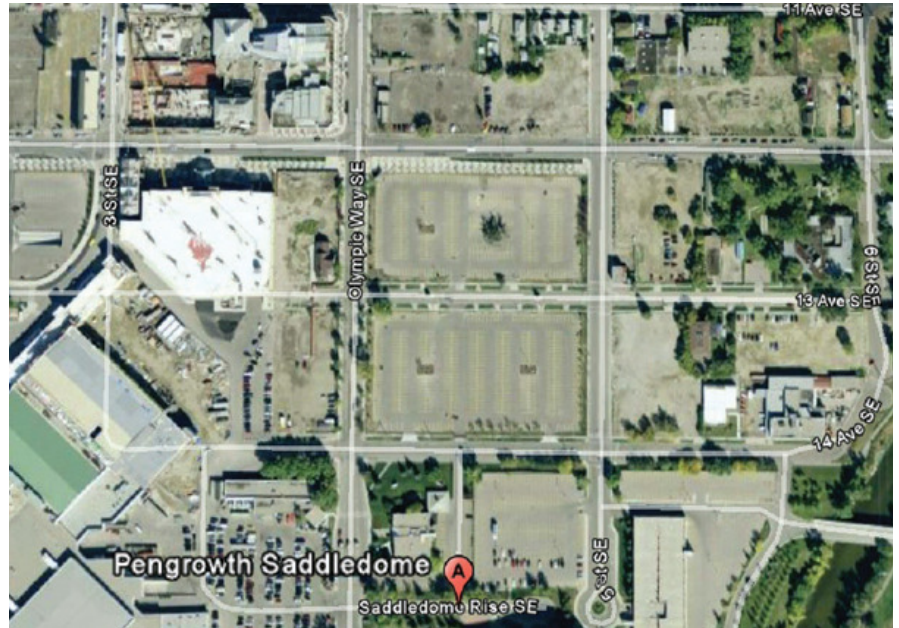
Make Connections

Look at the map.
Which streets are parallel
to 11th Avenue?
Which streets are perpendicular
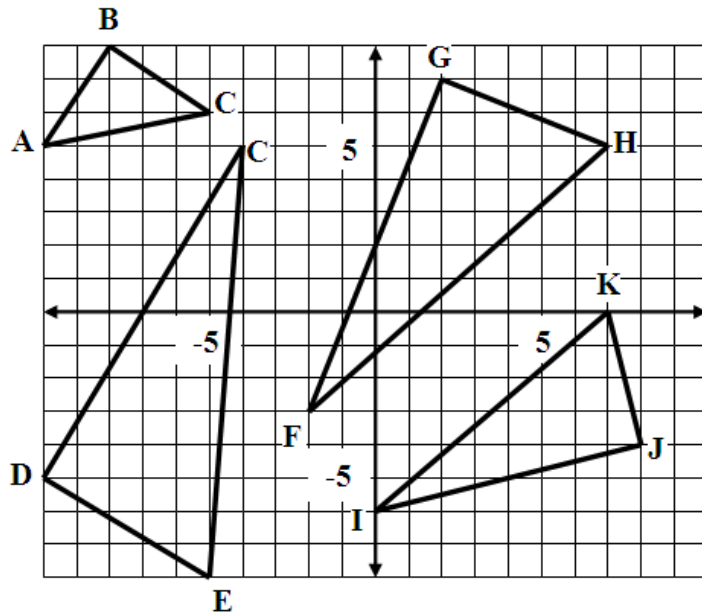to 11th Avenue?
How could you verify this?

Investigate:

Consider the following four right triangles, ΔABC, ΔCDE, ΔFGH, & ΔIJK

Complete the chart

Slope of	⊥ slope line segment
AB	BC
CD	DE
FG	GH
IJ	JK

What is the relationship between the slopes of the perpendicular lines segment

above? _____

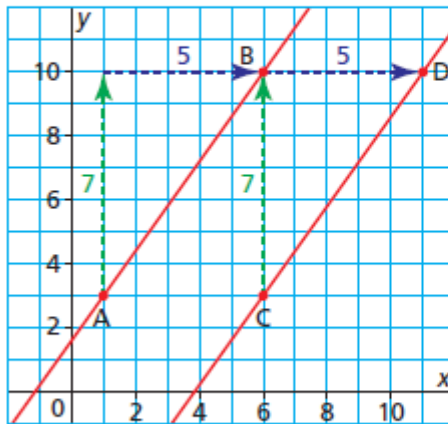**If the slopes of two line segments are** _____ **, the segments are perpendicular.**

If two line segments are perpendicular (and neither one is vertical), their

slopes are _____.

6

When two lines have the same slope, congruent triangles can be drawn to show the rise and the run.

Lines that have the some slope are parallel.

Slope of AB $= \dfrac{7}{5}$

Slope of CD $= \dfrac{7}{5}$

Since the slope of AB is equal to the slope of CD, line AB is parallel to line CD.

Example 1: Identifying Parallel Lines

Line GH passes through G($-4, 2$) and H($2, -1$). Line JK passes through J($-1, 7$) and K($7, 3$). Line MN passes through M($-4, 5$) and N($5, 1$). Sketch the lines. Are they parallel? Justify the answer.

CHECK YOUR UNDERSTANDING

1. Line EF passes through E($-3, -2$) and F($-1, 6$). Line CD passes through C($-1, -3$) and D($1, 7$). Line AB passes through A($-3, 7$) and B($-5, -2$). Sketch the lines. Are they parallel? Justify your answer.

[Answer: The slopes of the lines are not equal, so the lines are not parallel.]
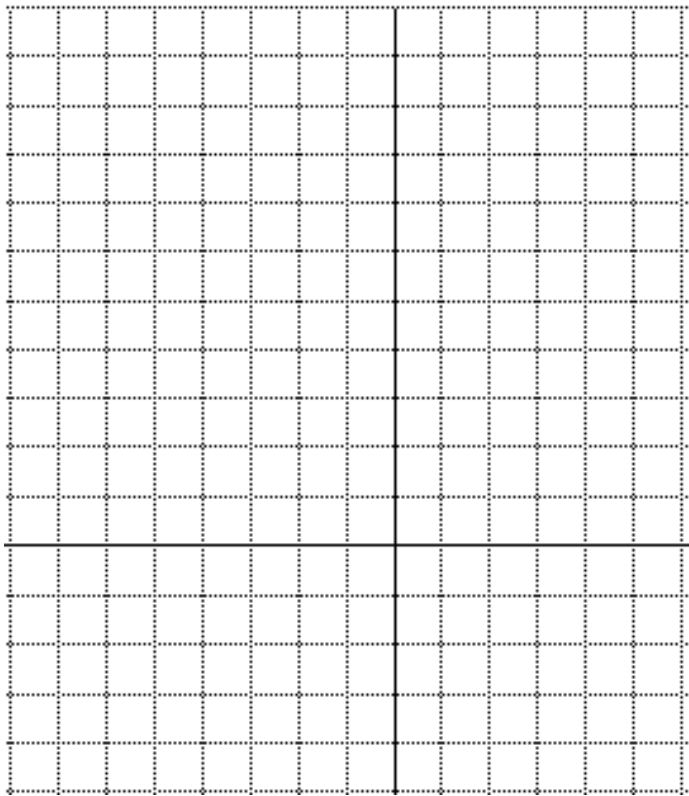
7
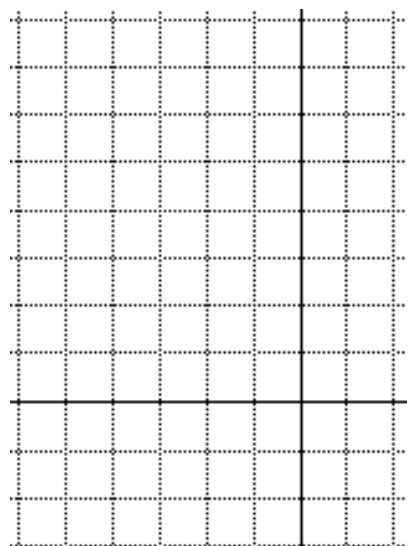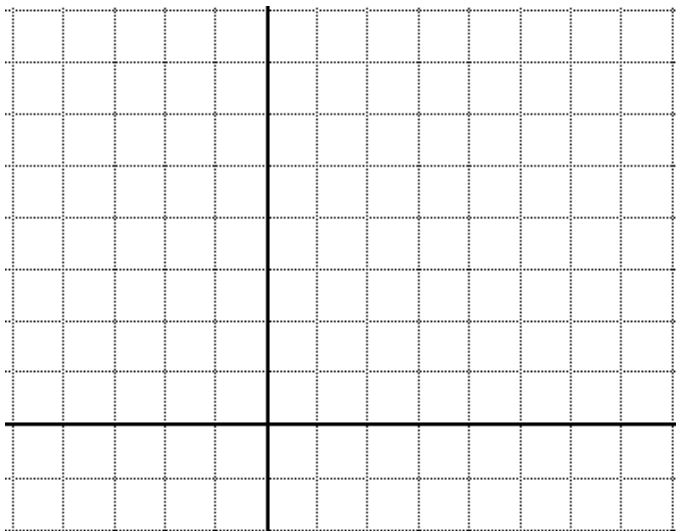
Example 2: Examining Slopes to Compare Lines

Line PQ passes through P$(-7, 2)$ and Q$(-2, 10)$.
Line RS passes through R$(-3, -4)$ and S$(5, 1)$.

a) Are these two lines parallel, perpendicular, or neither? Justify the answer.
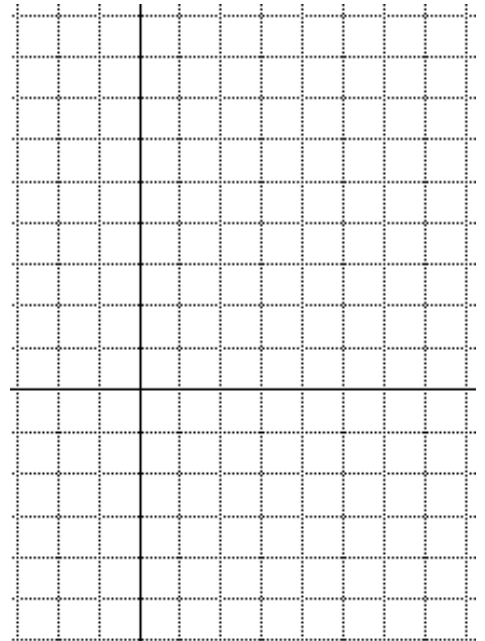
b) Sketch the lines to verify the answer to part a.

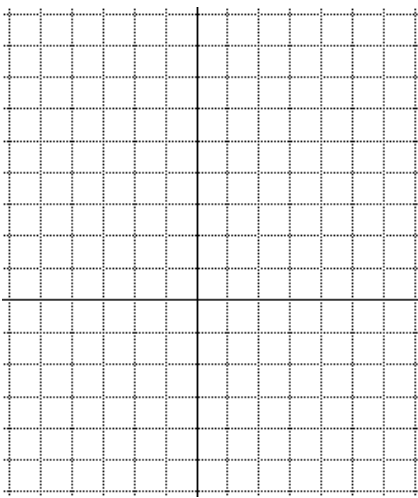## Example 3: Identifying a Line Perpendicular to a Given Line

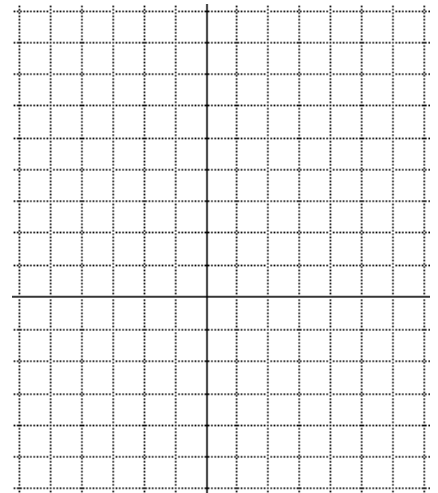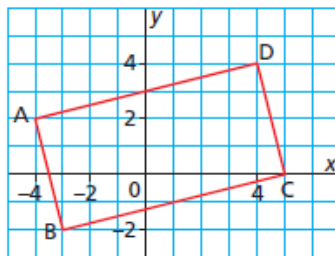a) Determine the slope of a line that is perpendicular to the line through E(2, 3) and F(−4, −1).

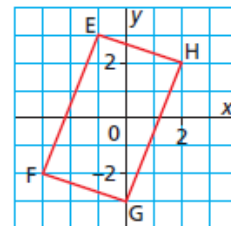b) Determine the coordinates of G so that line EG is perpendicular to line EF.

Example 4: Using Slope to Identify a Polygon

ABCD is a parallelogram. Is it a rectangle? Justify the answer.

Homework: Page 349   #6, 8–11, 14, 16, 18

Connections

Concept Development
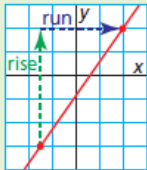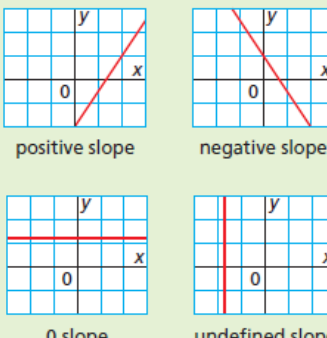
Definition

The slope of a line is the measure of its rate of change.

$$Slope = \frac{rise}{run}$$

positive slope

negative slope

0 slope

undefined slope
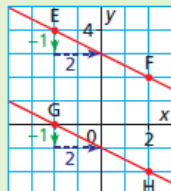
The slope of a line through $P(x_1, y_1)$ and $Q(x_2, y_2)$ is:

$$Slope = \frac{y_2 - y_1}{x_2 - x_1}$$
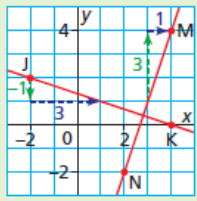

Two lines are parallel when they have equal slopes.

Slope of EF $= -\frac{1}{2}$

Slope of GH $= -\frac{1}{2}$

Two lines are perpendicular when their slopes are negative reciprocals.

Slope of MN $= 3$

Slope of JK $= -\frac{1}{3}$

$(3)\left(-\frac{1}{3}\right) = -1$

■ **In Lesson 6.1**

– You defined the slope of a line segment and the slope of a line as rate of change.

– You determined the slope of a line segment and the slope of a line from measurements of the rise and run.

– You showed that the slope of a line is equal to the slope of any segment of the line.

– You determined the slope of a line segment given the coordinates of the endpoints of the segment, and the slope of a line given the coordinates of two points on the line.

– You explained the meaning of the slope of a horizontal line and a vertical line.

– You drew a line, given its slope and a point on the line.

– You determined the coordinates of a point on a line, given its slope and another point on the line.

– You solved contextual problems involving slope.

■ **In Lesson 6.2**

– You generalized and applied rules for determining whether two lines are parallel or perpendicular.

– You drew lines that were parallel or perpendicular to a given line.

6.3 Math Lab: Investigating Graphs of Linear Functions

LESSON FOCUS:
Investigate the relationship between the graph and the equation of a linear function.
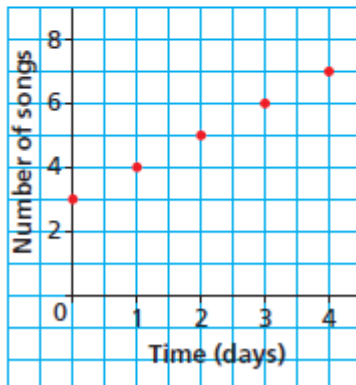

Make Connections

Alimina purchased an mp3 player and downloaded 3 songs.
Each subsequent day, she downloads 2 songs.
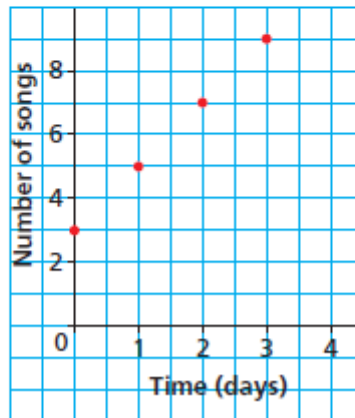Which graph represents this situation? Explain your choice.

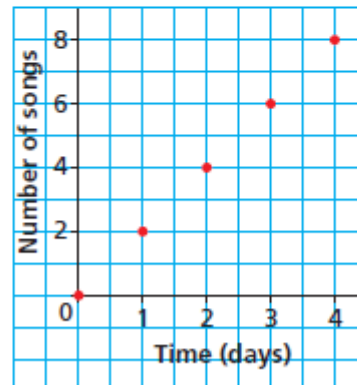

Graph A
Songs Downloaded to
an mp3 Player

Graph B
Songs Downloaded to
an mp3 Player

Graph C
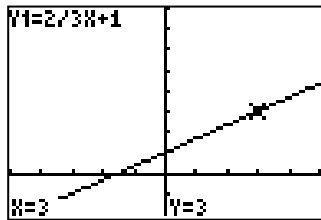Songs Downloaded to
an mp3 Player

Intercepts

x-intercept: the x-coordinate where the graph of the line intersects the x-axis.
(value of y is _____)

y-intercept: the y-coordinate where the graph of the line intersects the y-axis.
(value of x is _____)

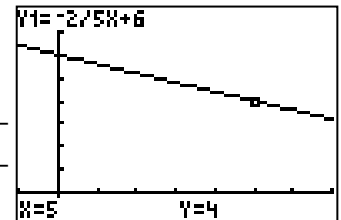**Investigate: Find the slope and y-intercept of each equation.**

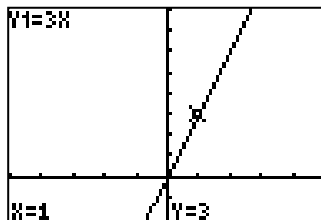1. $y = \dfrac{2}{3}x + 1$

   Slope = _____
   y-intercept = _____
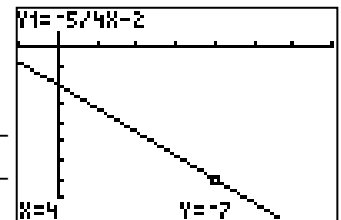
2. $y = 3x$

   Slope = _____
   y-intercept = _____

3. $y = \dfrac{7}{2}x - 3$
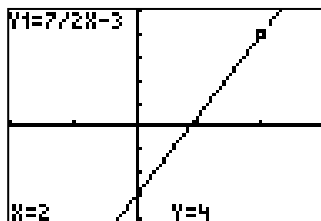
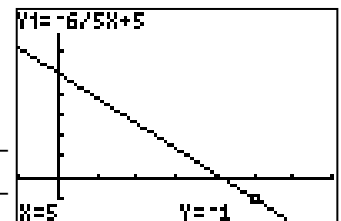   Slope = _____
   y-intercept = _____
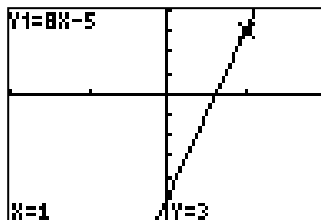
4. $y = 8x - 5$

   Slope = _____
   y-intercept = _____

5. $y = \dfrac{7}{2}x - 1$

   Slope = _____
   y-intercept = _____

6. $y = -\dfrac{4}{3}x + 7$

   Slope = _____
   y-intercept = _____

7. $y = -\dfrac{2}{5}x + 6$

   Slope = _____
   y-intercept = _____

8. $y = -\dfrac{5}{4}x - 2$

   Slope = _____
   y-intercept = _____

9. $y = -\dfrac{6}{5}x + 5$

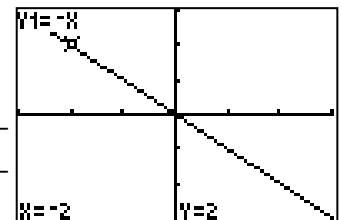   Slope = _____
   y-intercept = _____

10. $y = -x$

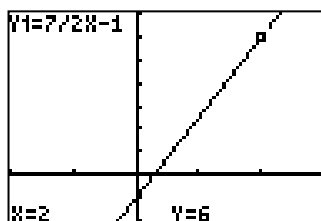   Slope = _____
    y-intercept = _____

11. $y = 3x + 2$

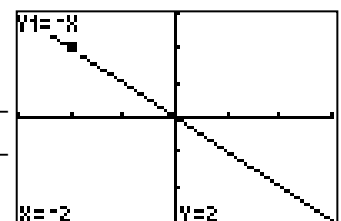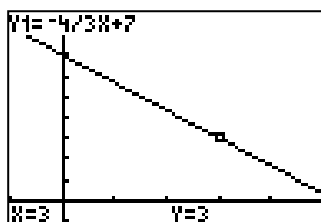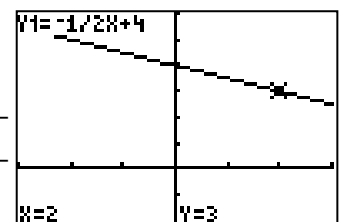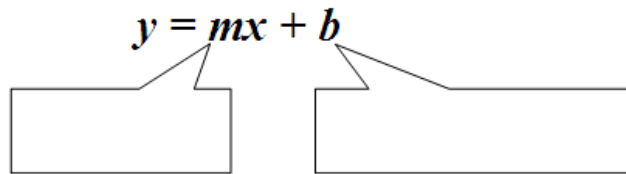   Slope = _____
    y-intercept = _____

12. $y = -\dfrac{1}{2}x + 4$

    Slope = _____
    y-intercept = _____

12

The graph of the equation $y = mx + b$ is a straight line with slope _____ and

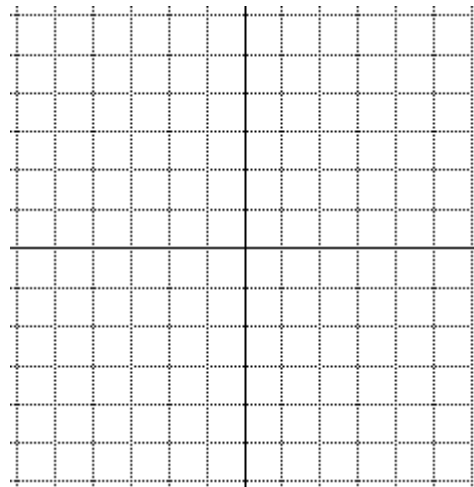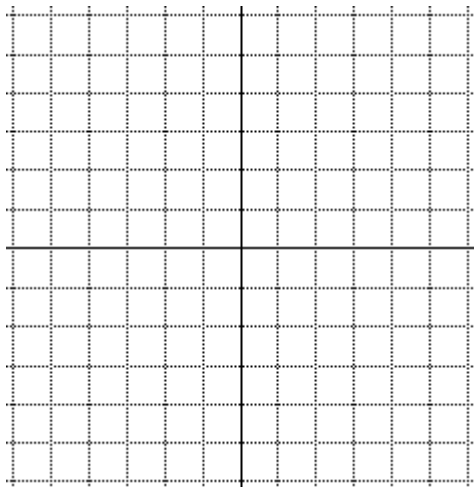y-intercept _____.

$$y = mx + b$$

The equation $y = mx + b$ is called the *slope y-intercept form* of the equation of the line.

Examples:

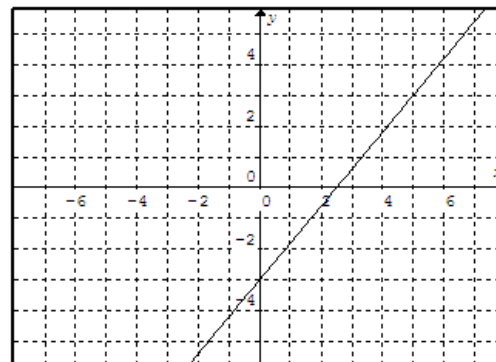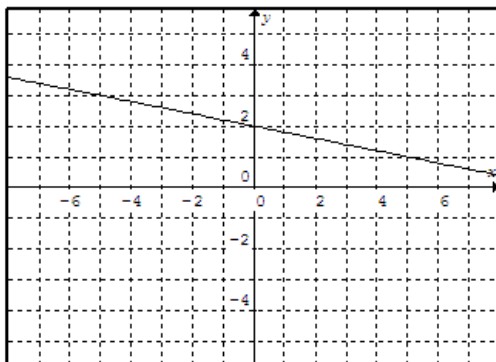1. Graph the following lines without using a table of values.

   a) $y = -\dfrac{3}{5}x + 3$

   b) $y = 3x - 2$

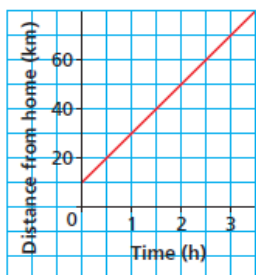2. Determine the equation of each line.

Homework: Page 356   #1–7

6.4 Slope-Intercept Form of the Equation for a Linear Function

LESSON FOCUS: Relate the graph of a linear function to its equation in slope-intercept form.

Make Connections

This graph shows a cyclist's journey where the distance is measured from her home.
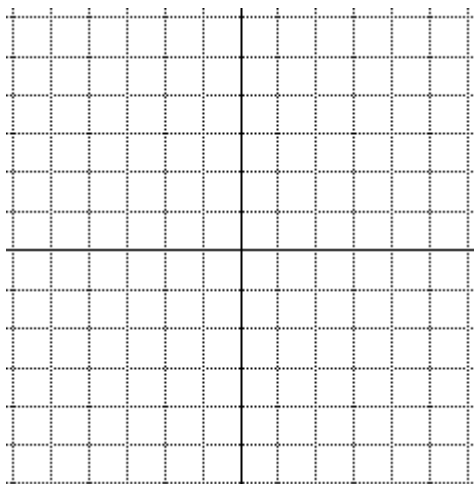
Graph of a Bicycle Journey

- What does the vertical intercept represent?

- What does the slope of the line represent?

Example 1: Writing an Equation of a Linear Function Given Its Slope and y-Intercept

The graph of a linear function has slope $\frac{3}{5}$ and y-intercept -4.

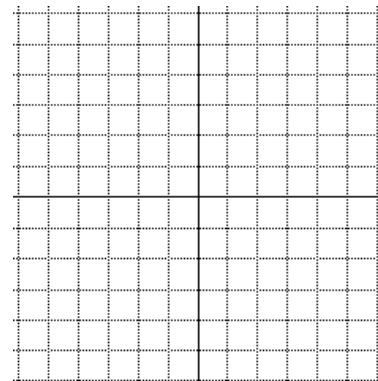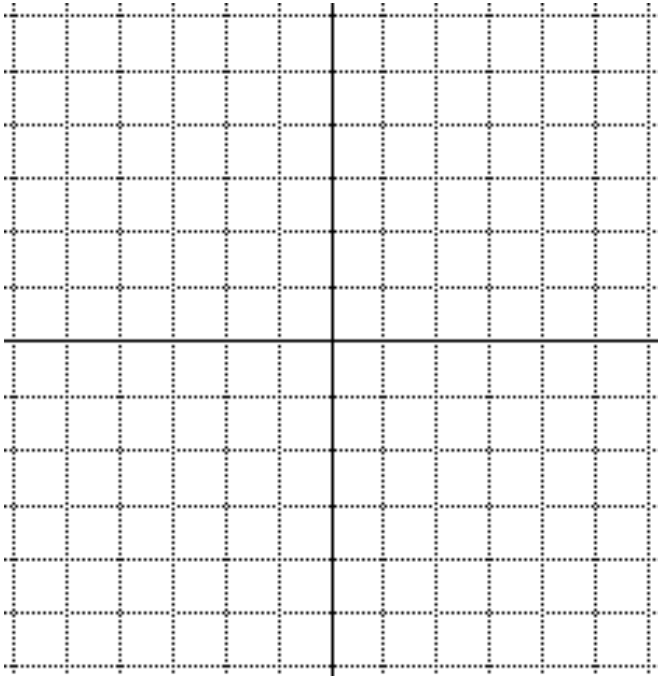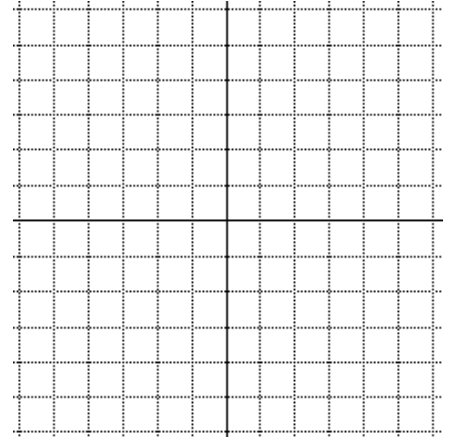Write an equation for this function.

14

Example 2: Graphing a Linear Function Given Its Equation in Slope-Intercept Form

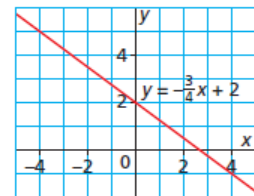Graph the linear function with equation: $y = \frac{1}{2}x + 3$

Example 3: Writing the Equation of a Linear Function Given Its Graph

Write an equation to describe this function. Verify the equation.

$y = f(x)$

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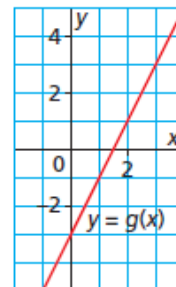
Example 4: Using an Equation of a Linear Function to Solve a Problem

The student council sponsored a dance. A ticket cost $5 and the cost for the DJ was $300.

a) Write an equation for the profit, P dollars, on the sale of t tickets.

b) Suppose 123 people bought tickets. What was the profit?

c) Suppose the profit was $350. How many people bought tickets?

d) Could the profit be exactly $146? Justify the answer.

Homework: Page 362   #7, 8, 12, 14, 18, 19

6.5 Slope-Point Form of the Equation for a Linear Function

LESSON FOCUS
Relate the graph of
a linear function to
its equation in
slope-point form.

Make Connections

This graph shows the height of a candle as it burns.
How would you write an equation to describe this line?
Suppose you could not identify the *h*-intercept.
How could you write an equation for the line?
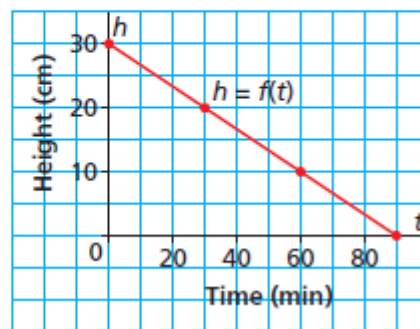
Developing Slope-Point Form of the Equation for a Linear Function
This line has slope *m* and passes through the point P(x_1, y_1).
Another point on the line is Q(*x*, *y*).

Example 1: Graphing a Linear Function Given Its Equation in Slope-Point Form

a) Describe the graph of the linear function with this equation:

$$y - 2 = \frac{1}{3}(x + 4)$$

b) Graph the equation.

Example 2: Writing an Equation Using a Point on the Line and Its Slope

a) Write an equation in slope-point form for this line.
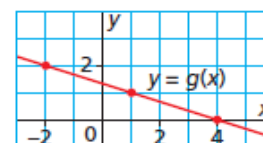
b) Write the equation in part a in slope-intercept form. What is the y-intercept of this line?

$y = f(x)$

Example 3: Writing an Equation of a Linear Function Given Two Points

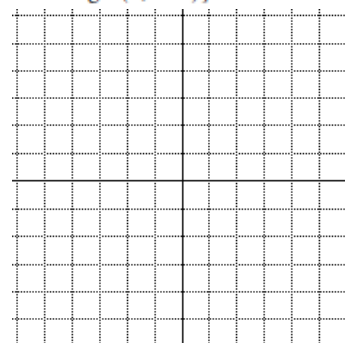The sum of the angles, s degrees, in a polygon is a linear function of the number of sides, n, of the polygon. The sum of the angles in a triangle is 180°. The sum of the angles in a quadrilateral is 360°.

a) Write a linear equation to represent this function.

b) Use the equation to determine the sum of the angles in a dodecagon.

Example 4: Writing an Equation of a Line That Is Parallel or Perpendicular to a Given Line

Write an equation for the line that passes through R$(1, -1)$ and is:

a) parallel to the line $y = \frac{2}{3}x - 5$

b) perpendicular to the line $y = \frac{2}{3}x - 5$

Homework: Page 372   #5–14, 21, 22

CHECKPOINT 2

Connections

The situation involves a constant rate of change and an initial value.

I know the slope and the y-intercept.

Slope-Intercept Form
$y = mx + b$

Linear Relations

Slope-Point Form
$y - y_1 = m(x - x_1)$

I know two points on the graph.

I know the slope and one point on the graph.

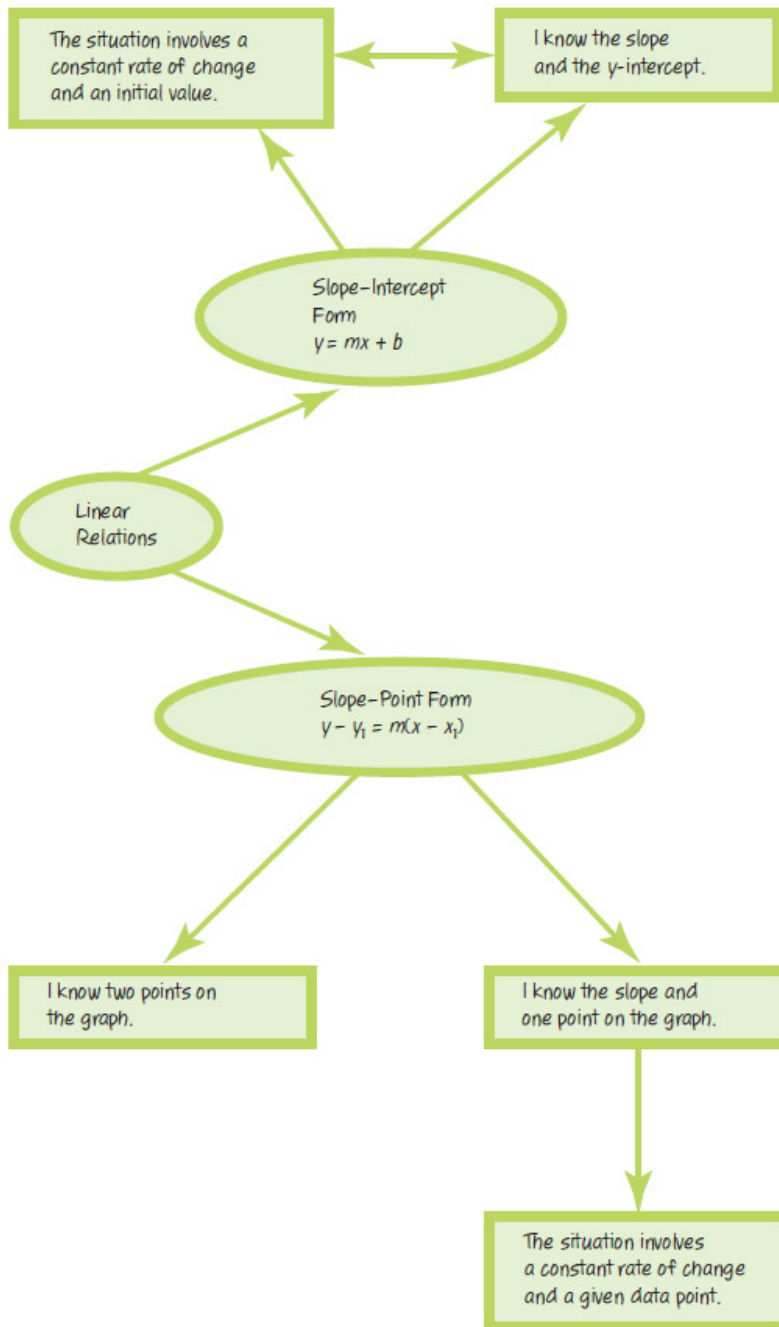The situation involves a constant rate of change and a given data point.

Concept Development

- **In Lesson 6.3**
  - You used technology to explore how changes in the constants m and b in the equation $y = mx + b$ affect the graph of the function.

- **In Lesson 6.4**
  - You used the slope and y-intercept of the graph of a linear function to write the equation of the function in slope-intercept form.
  - You graphed a linear function given its equation in slope-intercept form.
  - You used the graph of a linear function to write an equation for the function in slope-intercept form.

- **In Lesson 6.5**
  - You developed the slope-point form of the equation of a linear function.
  - You graphed a linear function given its equation in slope-point form.
  - You wrote the equation of a linear function after determining the slope of its graph and the coordinates of a point on its graph.
  - You wrote the equation of a linear function given the coordinates of two points on its graph.
  - You rewrote the equation of a linear function from slope-point form to slope-intercept form.

6.6 General Form of the Equation for a Linear Relation

LESSON FOCUS
Relate the graph of a linear function to its equation in general form.

Make Connections

A softball team may field any combination of 9 female and male players. There must be at least one female and one male on the field at any time. What are the possible combinations for female and male players on the field?

General Form of the Equation of a Linear Relation

$Ax + By + C = 0$ is the general form of the equation of a line, where A is a whole number, and B and C are integers.

What values of A, B, C, would produce a vertical line? A horizontal line?

Example 1: Rewriting an Equation in General Form

Write each equation in general form.

a) $y = -\frac{2}{3}x + 4$

b) $y - 1 = \frac{3}{5}(x + 2)$
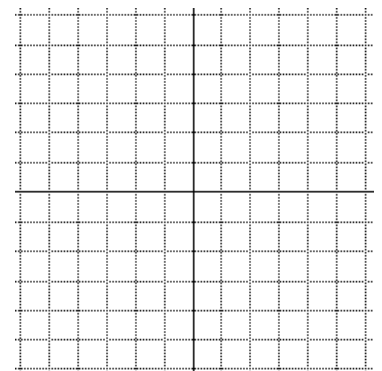
Example 2: Graphing a Line in General Form

a) Determine the x- and y-intercepts of the line whose equation is: $3x + 2y - 18 = 0$

b) Graph the line.

c) Verify that the graph is correct.

Example 3: Determining the Slope of a Line Given Its Equation in General Form

Determine the slope of the line with this equation:

$3x - 2y - 16 = 0$

Example 4: Determining an Equation from a Graph of Generated Data

Peanuts cost $2 per 100 g and raisins cost $1 per 100 g. Devon has $10 to purchase both these items.

a) Generate some data for this relation.

b) Graph the data.

c) Write an equation for the relation in general form.

d) i) Will Devon spend exactly $10 if she buys 300 g of peanuts and 400 g of raisins?

   ii) Will Devon spend exactly $10 if she buys 400 g of peanuts and 300 g of raisins?

Use the graph and the equation to justify the answers.

Homework: Page 384   #8–10, 12, 18, 24

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SKILLS SUMMARY

Skill	Description	Example
Determine the slope of a line and identify parallel lines and perpendicular lines. [6.1, 6.2]	A line that passes through $P(x_1, y_1)$ and $Q(x_2, y_2)$ has slope, m, where: $$m = \frac{y_2 - y_1}{x_2 - x_1}$$	For $P(-2, 4)$ and $Q(2, -1)$: $$m = \frac{-1 - 4}{2 - (-2)}, \text{ or } -\frac{5}{4}$$ The slope of a line parallel to PQ is $-\frac{5}{4}$. The slope of a line perpendicular to PQ is $\frac{4}{5}$.
Write the equation of a line in slope-intercept form. [6.4]	A line with slope, m, and y-intercept, b, has equation: $$y = mx + b$$	For a line with slope $-\frac{5}{4}$ and y-intercept 3, an equation is: $$y = -\frac{5}{4}x + 3$$
Write the equation of a line in slope-point form. [6.5]	A line with slope, m, and passing through $P(x_1, y_1)$, has equation: $$y - y_1 = m(x - x_1)$$	A line with slope $\frac{4}{5}$ and passing through $P(-2, 4)$ has equation: $$y - 4 = \frac{4}{5}(x - (-2)), \text{ or }$$ $$y - 4 = \frac{4}{5}(x + 2)$$
Graph a linear relation in general form. [6.6]	The general form of the equation is: $$Ax + By + C = 0$$ Determine intercepts by substituting: $x = 0$, and solving for y, then $y = 0$, and solving for x. Plot points at the intercepts, then draw a line through the points.	A line has equation: $3x + 4y + 12 = 0$ For the y-intercept: $$3(0) + 4y + 12 = 0$$ $$4y = -12$$ $$y = -3$$ For the x-intercept: $$3x + 4(0) + 12 = 0$$ $$3x = -12$$ $$x = -4$$

Do Review Pages 388 – 390   #1 – 28