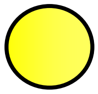
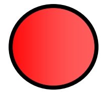
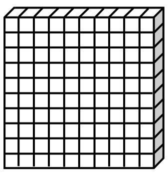
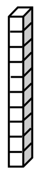
5.1 – Modelling Polynomials

Focus: Learn vocabulary associated with polynomials and represent and identify polynomials using models

Recall: Last year when we added and subtracted integers, we used integer chips.

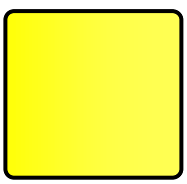
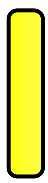
 Value: ______  Value: ______

In Arithmetic we used 10 blocks to model whole numbers.

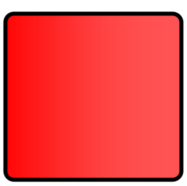
 Value: ______  Value: ______  Value: ______

To **model Polynomials** we will use **algebra tiles.**

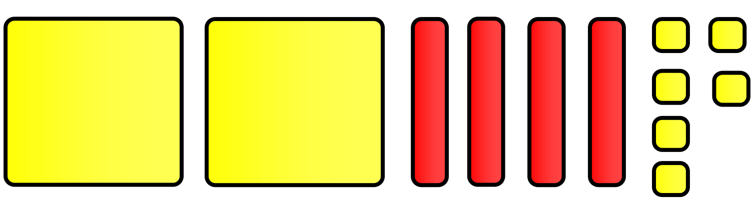
Yellow represents **positive** tiles.

 Value: ______  Value: ______  Value: ______

Red represents **negative** tiles.

 Value: ______  Value: ______  Value: ______

Ex. 1: The following tiles represent what expression?



Do the 'Investigate'

Investigate

Use algebra tiles.

➤ Model each expression. Sketch the tiles.
How do you know which tiles to use?
How do you know how many of each tile to use?

- $x^2 + x - 3$
- $-2x^2 - 3$
- $2x^2 + 3x$
- $-2x^2 - 3x + 1$
- $-3x + 3$

➤ Write your own expression.
Have your partner model it with tiles.
Model your partner's expression with tiles.

We must be able to identify vocabulary associated with Polynomials.

Let us examine the expression, $2x^2 - 4x + 12$.

1. What is a **variable?**

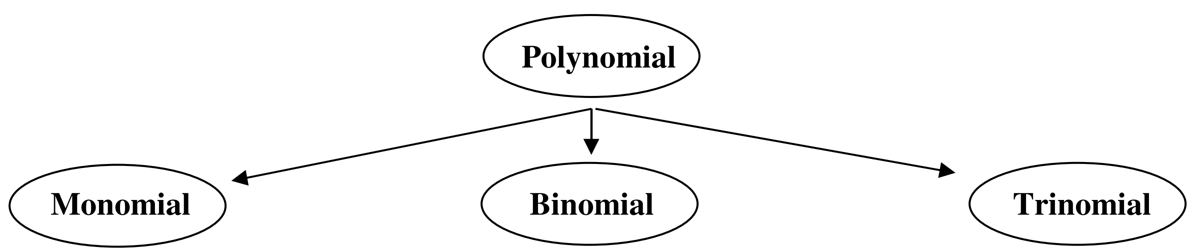
2. What is a **Coefficient?**

3. What is a **Constant?**

4. What are **terms**?

5. What is a **Polynomial**?

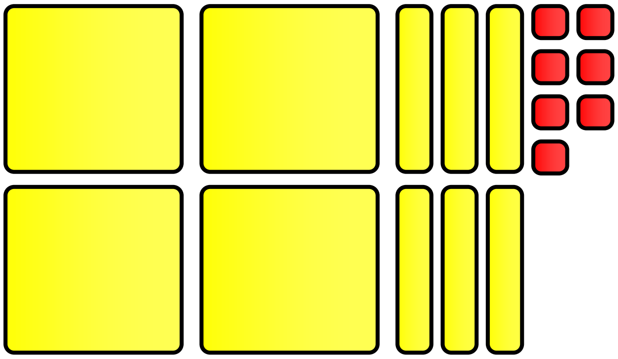
6. What is the **degree** of the polynomial?



A polynomial is usually written in descending order, meaning the exponents of the variable decrease from left to right.

An algebraic expression that contains a term with a variable in the denominator such as $\frac{5}{x}$, or the square root of a variable, such as $\sqrt{x}$ is **not** a polynomial.

Ex. 2: State the polynomials modeled by the algebra tiles and also the following:
i. the Degree, *ii.* The Constant term, *iii.* The Type of polynomial (monomial, binomial, trinomial)

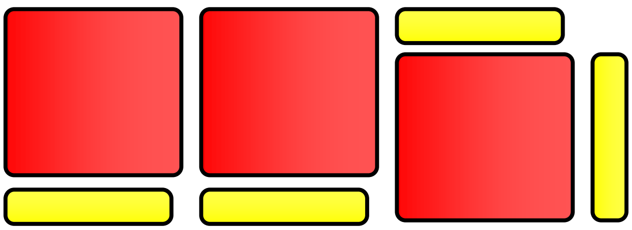
a) 

i. Degree: _______

ii. Constant: _______

iii. Type: __________________

Polynomial: __________________________

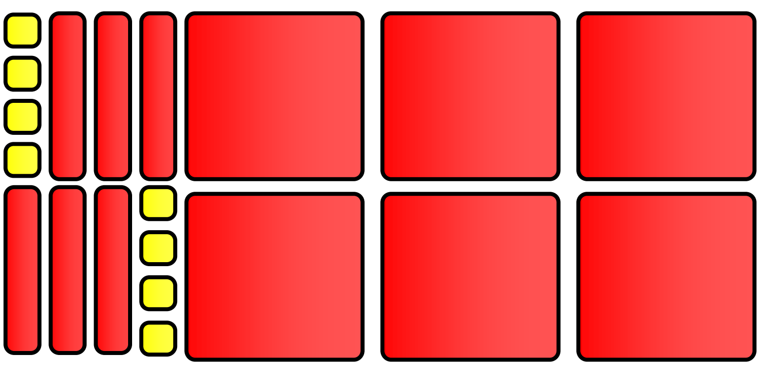
b) 

i. Degree: ______

*ii.*Constant: ______

iii. Type: __________________

Polynomial: __________________________

c) 

i. Degree: ______

*ii.*Constant: ______

iii. Type: __________________

Polynomial: __________________________

Ex. 3: Model the following polynomials using algebra tiles. State the degree and classify the polynomial (monomial, binomial, trinomial).

a) $4x+3$

degree: __________

classification: ________________________

b) $-5c$

degree: __________

classification: ________________________

c) $5a-4a^2-3$

degree: __________

classification: ________________________

d) $3y^2-5y$

degree: __________

classification: ________________________

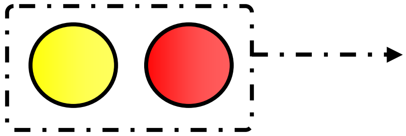
HW Assignment

Section 5.1 pg. 214 # 4, 5,7, 9 – 13, 15, 16, 18

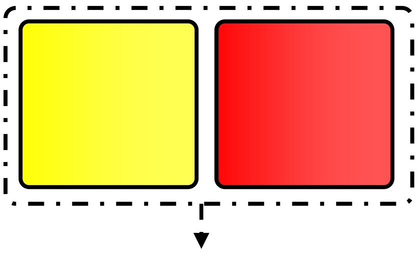
5.2 – Like Terms & Unlike Terms

Focus: Simplify polynomials by combining like terms

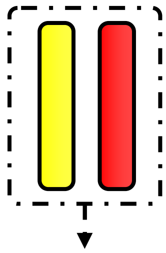
Recall: Last year when we added and subtracted integers we learned about the concept of **zero pairs**.

 Value: ______

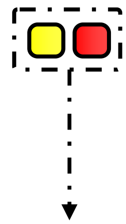
Predict the value of the following:

a) 

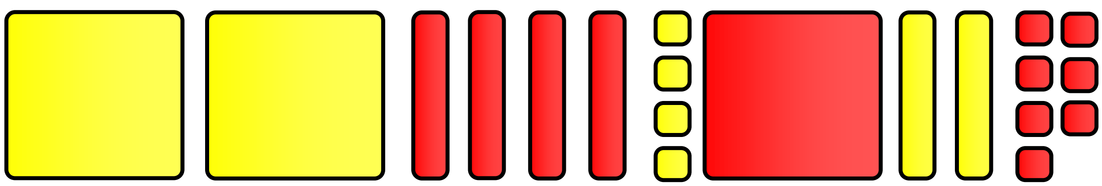
Value: ______

b) 

Value: ______

c) 

Value: ______

Ex. 1: 

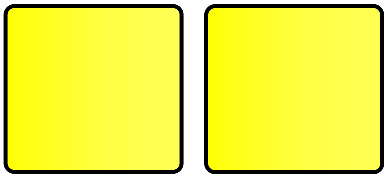
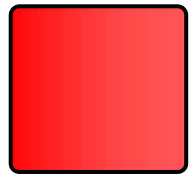
State the Polynomial: ____

a) Rearrange the tiles by organizing them according to their shapes.

b) Remove zero pairs.

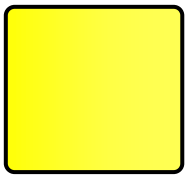
c) What polynomial remains?

Terms that can represented by tiles that are of the same size and shape are called **like terms.**

 ________  ________

Symbolically, terms that have the same variables, raised to the same exponents are called **like terms.**

Terms that are represented by tiles that are of the different size and shape are called **unlike terms.**

 ________  ________

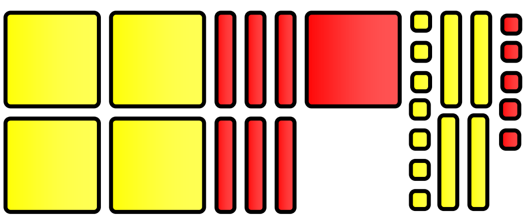
We **can not** combine unlike terms.

To simplify a polynomial, we group like terms and remove zero pairs.

To symbolically simplify a polynomial, combine like terms by adding the **coefficients** of like terms.

We can only combine like terms.

Ex. 2: Use Algebra tiles to simplify the following polynomial.



Polynomial: ______________________________

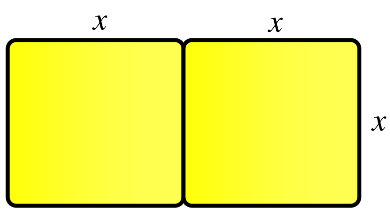
Tiles:

Symbolically:

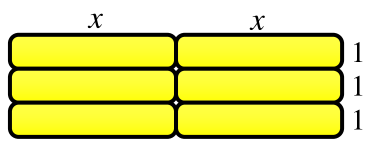
Ex. 3: Simplify: $12x^2 - 13 - 5x + 6 - 10x - 16x^2$

Ex. 4: Write a polynomial to represent the perimeter of each rectangle.

a)



b)



Ex. 5: Each polynomial represents the perimeter of a rectangle.
Use algebra tiles to model the rectangle.

a) $6x+4$

b) $8x$

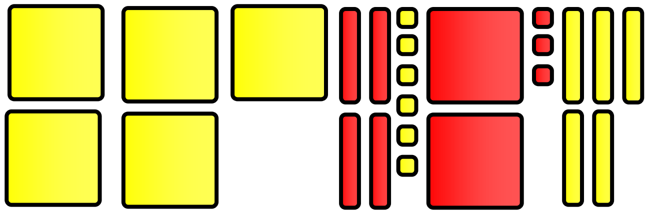
Ex. 6: Simplify: $3xy - y^2 + 4x - 5xy - 6y - 8y^2$

HW Assignment Section 5.2 pg. 222 # 6 – 8, 10, 11ace, 13, 14, 18a, 19, 20

5.3 – Adding Polynomials

Focus: Use different Strategies to Add polynomials

Recall:



State the simplified polynomial by:

a) Rearranging the algebra tiles according to their shapes.

b) Removing zero pairs.

c) Drawing the remaining tiles.

Simplified Polynomial: ________________________

Adding Polynomials

When we write the sum of two polynomials, we write each polynomial in brackets.

To add polynomials, we can use a few different methods.

1) *Using Algebra Tiles* 2) *Combine like terms by adding their coefficients*

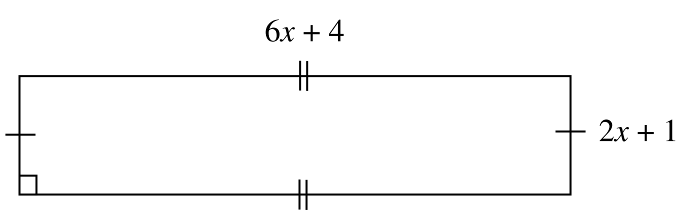
Ex. 1: Determine the sum of the polynomials $3x^2 + x - 4$ and $-4x^2 + 3x + 2$ using Algebra tiles.

Ex. 2: Add without using algebra tiles: $(8a-4)+(-12a^2-3a-13)$

When Adding polynomials, we can add horizontally like above or we can also align the polynomials according to like terms and add vertically.

Ex. 3: Add $(3x^2-4x+5y-8xy+4y^2)+(-6y-8x+7x^2-4xy-3y^2)$

Ex. 4: Write the Perimeter of the rectangle as a simplified polynomial.



HW Assignment
Section 5.3 pg. 228 # 3, 5 – 7, 8 – 9 (aceg), 10a, 12, 14, 15ace, 16, 17

5.4 – Subtracting Polynomials

Focus: Use different Strategies to Subtract polynomials

Recall: Last year we used counters to add and subtract integers.

Value: ______

Value: ______

Model the following differences using counters.

a) $5-3$ b) $-4-(-2)$ c) $-2-(-5)$ d) $6-(-2)$

In some cases there aren't enough counters or any at all to take away.
In these cases we **add zero pairs** according to the second integer and subtract as needed.

To subtract integers without using models, change from a subtraction question to an addition question and evaluate using your knowledge of integer addition.

To subtract polynomials we will use the above properties of integer subtraction.

Subtracting Polynomials

When we write the difference of two polynomials, we write each polynomial in brackets.

To subtract polynomials, we can use a few different methods.

1) *Using Algebra Tiles*
   - Model the first polynomial using tiles.
   - Take away tiles according to the second polynomial
   - If there are not enough or no tiles to take away, add zero pairs, then take the tiles away.

2) *Combine like terms by subtracting their coefficients.*
   - To subtract coefficients, change to addition by adding the opposite.

Ex. 1: Subtract the following using algebra tiles. $(4x^2-2x+4)-(-x^2+4x+3)$

Ex. 2: Solve using algebra tiles and symbolically. $(-3y^2+3y+4)-(2y^2+4y-6)$

Algebra tiles: Symbolically:

Ex. 3: Subtract $(2x^2-6x+4y-8xy+9y^2)-(-2y+3x+7x^2-5xy-4y^2)$

Check you answer using addition.

HW Assignment
Section 5.4 pg. 234 # 4, 6bc, 7 – 9, 12, 13, 15, 16
Quiz next class on 5.1 to 5.4

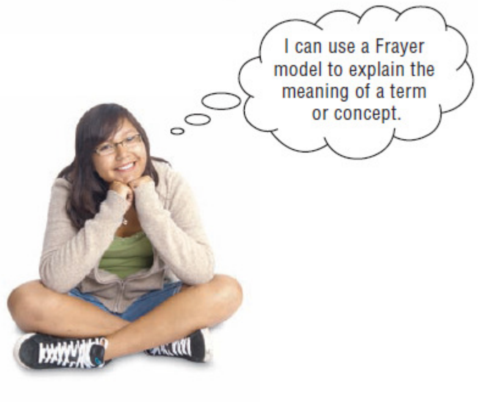
Start Where You Are

How Can I Summarize What I Have Learned?

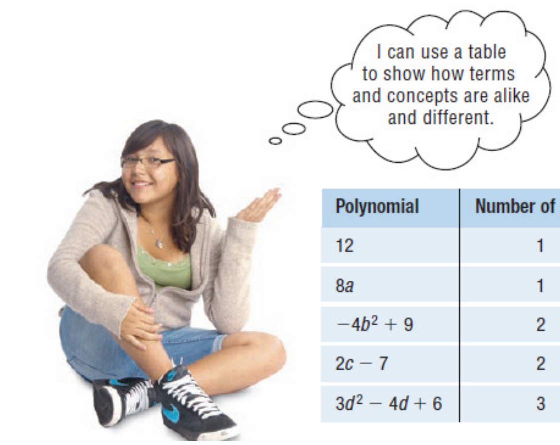
Suppose I want to summarize what I know about polynomials.

➤ What tools could I use to do this?

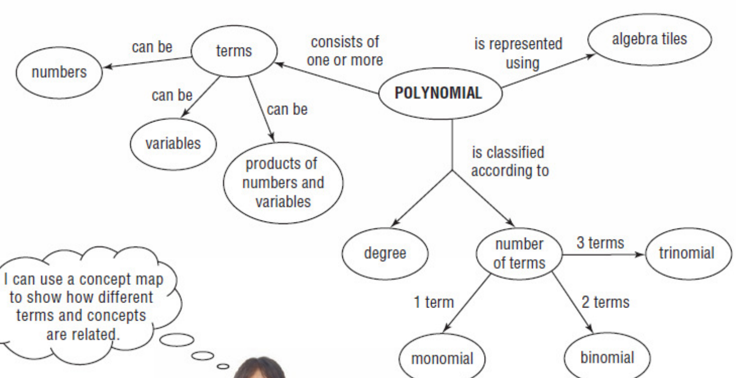
- a Frayer model
- a table
- a concept map



Definition Like terms have the same variable raised to the same exponent.	**Facts/Characteristics** Like terms are represented by algebra tiles with the same size and shape. I can combine like terms by adding their coefficients.
Like terms	
Examples $-3x$ and $4x$ $5b^2$ and $2b^2$	**Non-examples** $-3c$ and 4 $5n^2$ and $2n$



Polynomial	Number of Terms	Name by Number of Terms	Degree
12	1	monomial	0
$8a$	1	monomial	1
$-4b^2 + 9$	2	binomial	2
$2c - 7$	2	binomial	1
$3d^2 - 4d + 6$	3	trinomial	2



Mid-Unit Review

5.1

1. In each polynomial, identify: the variable, number of terms, coefficients, constant term, and degree.

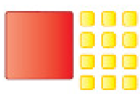
a) $3m - 5$

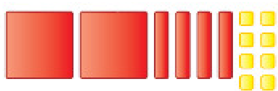
b) $4r$

c) $x^2 + 4x + 1$

2. Create a polynomial that meets these conditions: trinomial in variable m, degree 2, constant term is -5

3. Which polynomial is represented by each set of algebra tiles? Is the polynomial a monomial, binomial, or trinomial? How do you know?

a) 

b) 

c) 

4. Use algebra tiles to represent each polynomial. Sketch the tiles you used.

a) $4n - 2$

b) $-t^2 + 4t$

c) $2d^2 + 3d + 2$

5.2

5. For each pair of monomials, which are like terms? Explain how you know.

a) $2x, -5x$

b) $3, 4g$

c) $10, 2$

d) $2q^2, -7q^2$

e) $8x^2, 3x$

f) $-5x, -5x^2$

6. Simplify $3x^2 - 7 + 3 - 5x^2 - 3x + 5$. Explain how you did this.

7. Renata simplified a polynomial and got $4x^2 + 2x - 7$. Her friend simplified the same polynomial and got $-7 + 4x^2 + 2x$. Renata thinks her friend's answer is wrong. Do you agree? Explain.

8. Cooper thinks that $5x - 2$ simplifies to $3x$. Is he correct? Explain. Use algebra tiles to support your explanation.

9. Identify the equivalent polynomials. Justify your answers.

a) $1 + 3x - x^2$

b) $1 + 3x^2 - x^2 + 2x - 2x^2 + x - 2$

c) $x^2 - 3x - 1$

d) $6 + 6x - 6x^2 - 4x - 5 + 2x^2 + x^2 - 4$

e) $3x - 1$

f) $-3x^2 + 2x - 3$

g) $6x^2 - 6x - 6 + x - 5x^2 - 1 + 2x + 4$

h) $3x - x^2 + 1$

5.3
5.4

10. Use algebra tiles to add or subtract. Sketch the tiles you used.

a) $(4f^2 - 4f) + (-2f^2)$

b) $(3r^2 + 2r + 5) + (-7r^2 + r - 3)$

c) $(-2v + 5) - (-9v + 3)$

d) $(-2g^2 - 12) - (-6g^2 + 4g - 1)$

11. Add or subtract. Use a strategy of your choice.

a) $(3w^2 + 17w) + (12w^2 - 3w)$

b) $(5m^2 - 3) + (m^2 + 3)$

c) $(-3h - 12) - (-9h - 6)$

d) $(6a^2 + 2a - 2) + (-7a^2 + 4a + 11)$

e) $(3y^2 + 9y + 7) - (2y^2 - 4y + 13)$

f) $(-14 + 3p^2 + 2p) - (-5p + 10 - 7p^2)$

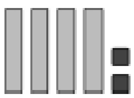
12. a) Which polynomial must be added to $5x^2 + 3x - 2$ to get $7x^2 + 5x + 1$?

b) Which polynomial must be subtracted from $5x^2 + 3x - 2$ to get $7x^2 + 5x + 1$?

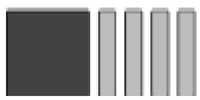
Justify your answers.

Answers:

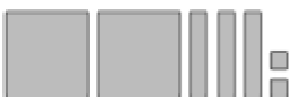
1. a) Variable: m; number of terms: 2; coefficient: 3; constant term: -5; degree: 1
   b) Variable: r; number of terms: 1; coefficient: 4; constant term: none; degree: 1
   c) Variable: x; number of terms: 3; coefficients: 1, 4; constant term: 1; degree: 2
2. Answers will vary, for example: $3m^2 - 4m - 5$
3. a) $-x^2 + 12$; binomial
   b) $-2x^2 - 4x + 8$; trinomial c) $-4x$; monomial
4. a)



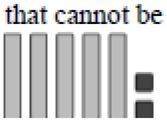
   b)



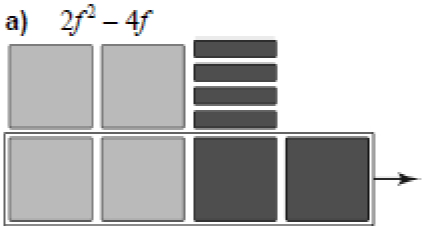
   c)



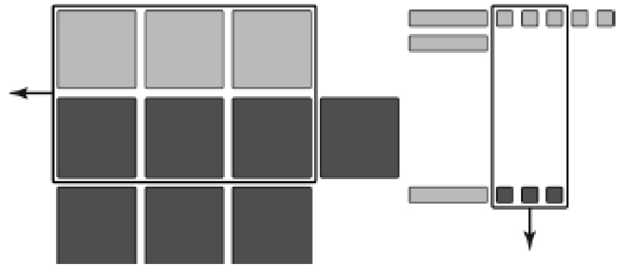
5. a) $2x$ and $-5x$ are like terms because they have the same variable raised to the same exponent.
   b) 3 and $4g$ are unlike terms because one is a constant and the other has a variable.
   c) 10 and 2 are like terms because they are both constants.
   d) $2q^2$ and $-7q^2$ are like terms because they have the same variable raised to the same exponent.
   e) $8x^2$ and $3x$ are unlike terms because they have variables raised to different exponents.
   f) $-5x^2$ and $-5x$ are unlike terms because they have variables raised to different exponents.
6. $-2x^2 - 3x + 1$
7. No, both answers are correct. The polynomials have their terms ordered differently.
8. No, Cooper is incorrect. $5x$ and -2 are unlike terms that cannot be simplified.



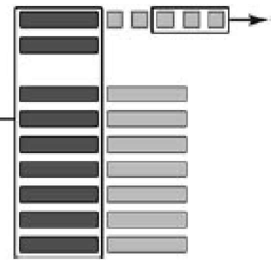
9. Parts a and h, b and e, d and f are equivalent.
10. a) $2f^2 - 4f$



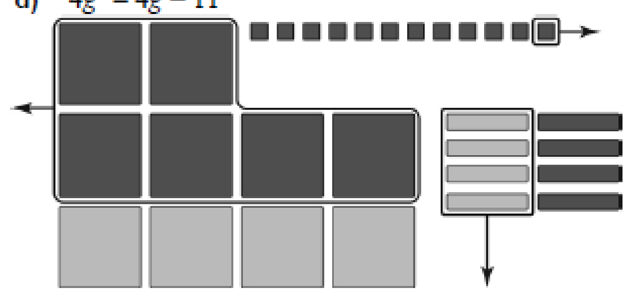
   b) $-4r^2 + 3r + 2$



   c) $7v + 2$



   d) $4g^2 - 4g - 11$

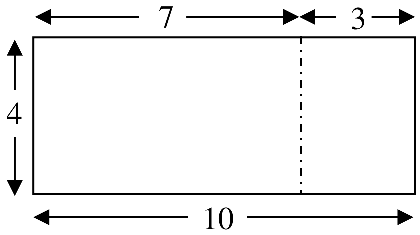


11. a) $15w^2 + 14w$ b) $6m^2$
    c) $6h - 6$ d) $-a^2 + 6a + 9$
    e) $y^2 + 13y - 6$ f) $10p^2 + 7p - 24$
12. a) $2x^2 + 2x + 3$ b) $-2x^2 - 2x - 3$

5.5 – Multiplying & Dividing a Polynomial by a Constant

Focus: Use different strategies to multiply and divide a polynomial by a constant.

Warm-up:



Calculate the Area of the Rectangle in two different ways.

1.

2.

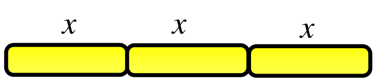
The diagram above represents the distributive property.

Distributive Property: $a(b+c) = ab + ac$

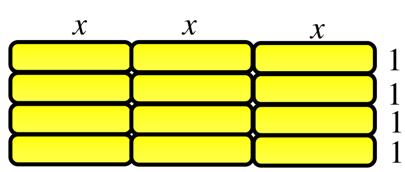
We will use the distributive property when we symbolically determine the product of a polynomial and a constant.

Multiplying a Polynomial by a Constant: *Algebra tiles.*

Model $3x$ using algebra tiles.



Use the previous model to model $4(3x)$.



____

Model $-2x$ using algebra tiles.

Use the previous model to model $3(-2x)$.

____

Model $2x-3$ using algebra tiles.

Use the previous model to model $4(2x-3)$

____

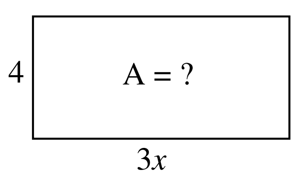
Model $2x-3$ using algebra tiles.

Use the previous model to model $-4(2x-3)$

Ex. 1: Determine the product of $-3(x^2-2x+4)$ using algebra tiles.

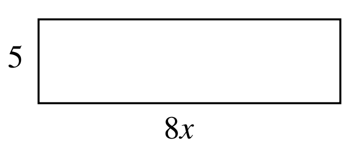
Multiplying a Polynomial by a Constant: *Area Model*

Determine the product of $4(3x)$.



Multiplying $4(3x)$ is equivalent to finding the area of a rectangle with dimensions, 4 and $3x$

Ex. 2: Find the Area of the rectangle.



Multiplying a Polynomial by a Constant: *Distributive Property*

To use the distributive property, multiply each term in the brackets by the term outside the brackets.

Ex. 3: Determine the product.

a) $5(-7x+4)$ b) $-2(-4x+12)$ c) $-3(-4x^2+6x-5)$

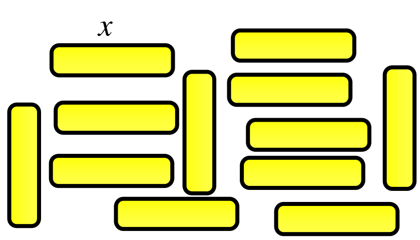
Multiplication and division are inverse operations.

To divide a polynomial by a constant, we reverse the process of multiplication.

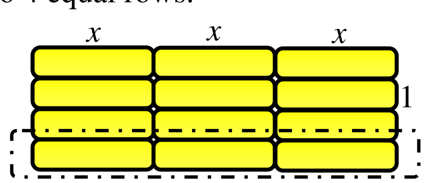
Dividing a Polynomial by a Constant: *Algebra tiles.*

Let us examine the quotient $12x \div 4$

Begin with 12 tiles that represent x.



Because we are dividing by four, organize the 12 tiles into 4 equal rows.



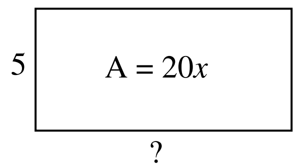
The number of tiles in each row represents the quotient.

Ex. 4: Determine the quotient using algebra tiles..

a) $\dfrac{6x+12}{3}$ b) $\dfrac{4x^2-8x+6}{2}$

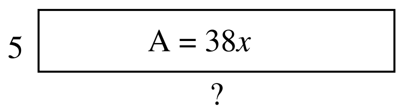
Dividing a Polynomial by a Constant: *Area Model*

Determine the quotient of $20x \div 5$



To determine the quotient we need to find the length of the missing side.

Ex. 5: Find the length of the missing side.



Dividing a Polynomial by a Constant

Dividing a polynomial by a constant is equivalent to dividing each term in the polynomial by the constant. So rewrite the quotient expression as a sum of fractions.

To divide a term by a constant, divide the coefficient by the constant.

Ex. 6: Determine the quotient.

a) $\dfrac{24x - 15}{3}$ b) $\dfrac{-6x^2 - 8x + 10}{2}$ c) $\dfrac{-8x^2 + 16x - 32}{-4}$

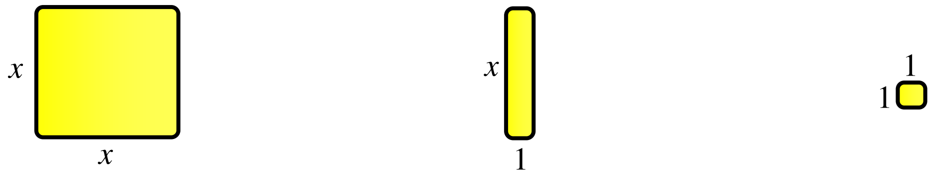
HW Assignment

Section 5.5 pg. 246 # 3, 5a, 6, 7a, 8a, 9, 11, 12, 13ace, 14 – 16, 17ace, 20, 22, 23

5.6 – Multiplying & Dividing a Polynomial by a Monomial

Focus: Use different strategies to multiply and divide a polynomial by a monomial

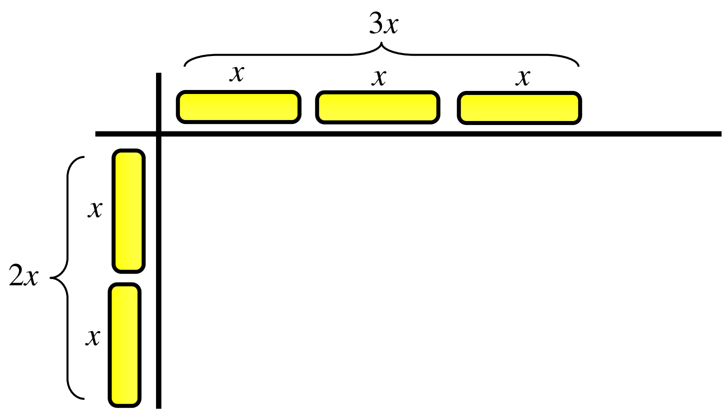
Recall the dimensions of the algebra tiles.



Multiplying a Polynomial by a Monomial: *Algebra tiles.*

Let us examine the product of the two monomials, $(3x)(2x)$.

We take the each to represent the dimensions of a rectangle made from algebra tiles.



What tiles do we need to build a rectangle with dimensions $3x$ and $2x$?

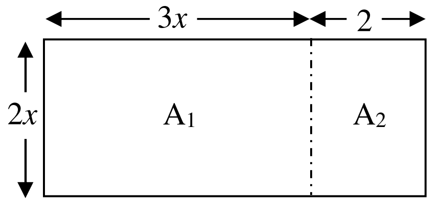
*Remember the sign rules when multiplying and dividing.

Ex. 1: Find the product of $-2x(3x+2)$ using algebra tiles.

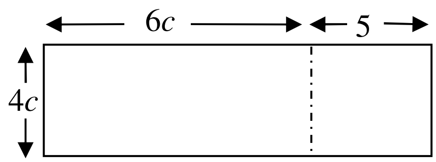
Multiplying a Polynomial by a Monomial: *Area Model*

Determine the product of $2x(3x+2)$.

Draw a rectangle with dimensions $2x$ and $3x + 2$.
Find the sum of the two areas.



Ex. 2: Write the multiplication sentence modeled by the rectangle.



Multiplying a Polynomial by a Monomial: *Distributive Property*

To multiply a polynomial by a monomial, we will use the distributive property and exponent laws.

Ex. 3: Determine the product.

a) $6m(4m-5)$ b) $-4x(3x-2y)$ c) $-3x(2x-4y+6)$

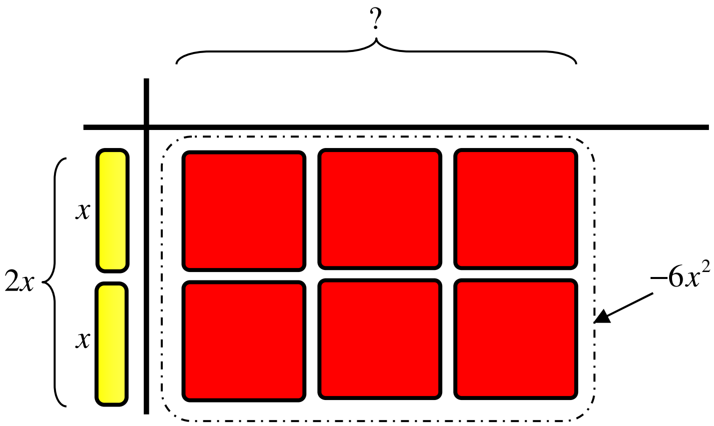
Multiplication and division are inverse operations.

To divide a polynomial by a monomial, we reverse the process of multiplication.

Dividing a Polynomial by a Monomial: *Algebra tiles.*

Let us examine the quotient $-6x^2 \div 2x$

The first polynomial is the number of tiles in the rectangle and the second number is one of the dimensions.



What is the unknown dimension?

Ex. 4: Find the quotient of $\dfrac{8x^2 - 12x}{4x}$ using algebra tiles.

Ex. 5: Find the quotient of $\dfrac{-8x^2 + 4x}{-2x}$ using algebra tiles.

Dividing a Polynomial by a Monomial

Dividing a polynomial by a monomial is equivalent to dividing each term in the polynomial by the monomial. So rewrite the quotient expression as a sum of fractions.

To divide a term by a monomial, divide the coefficients and divide the variables.

Ex. 6: Determine the quotient.

a) $\dfrac{27x}{-3x}$ b) $\dfrac{-12x^2 + 16x}{-4x}$ c) $\dfrac{24x^2 - 32xy}{-4x}$

HW Assignment

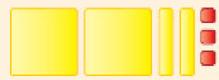
Section 5.6 pg. 255 # 4 – 7, 9a, 10a, 11, 12, 14, 16, 19 – 22

Study Guide

Polynomials

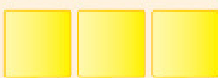
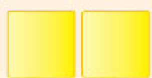
- A polynomial is one term or the sum of terms whose variables have whole-number exponents; for example, $2m^2 + 3m - 5$
- The numerical value of a term is its coefficient.
- A term that consists of only a number is a constant term.
- The degree of a polynomial in the variable m is the highest power of m in the polynomial.
- A polynomial with: 1 term is a monomial; 2 terms is a binomial; and 3 terms is a trinomial.

Algebra Tiles

We can represent a polynomial with algebra tiles. $2p^2 + 2p - 3$ 

Like Terms

Like terms are represented by the same type of algebra tile. In symbolic form, like terms have the same variables raised to the same exponent. Like terms can be added or subtracted. $3x^2$ and $2x^2$ are like terms, but $-x$ and 3 are not.

$3x^2$:  $2x^2$:  $3x^2 + 2x^2$ simplifies to $5x^2$.

$-x$:  3:  $-x + 3$ cannot be simplified.

Operations with Polynomials

We can use algebra tiles to model operations with polynomials, then record the answers symbolically.

- To add polynomials, combine like terms:
$$\begin{aligned}(3r^2 + 5r) + (2r^2 - r) &= 3r^2 + 5r + 2r^2 - r\\ &= 5r^2 + 4r\end{aligned}$$
- To subtract polynomials, use a strategy for subtracting integers:
$$\begin{aligned}(3r^2 + 5r) - (2r^2 - r) &= 3r^2 + 5r - (2r^2) - (-r)\\ &= 3r^2 - 2r^2 + 5r + r\\ &= r^2 + 6r\end{aligned}$$
- To multiply a polynomial by a monomial, multiply each term of the polynomial by the monomial:
$$\begin{aligned}2t(5t - 3) &= 2t(5t) + 2t(-3)\\ &= 10t^2 - 6t\end{aligned}$$
- To divide a polynomial by a monomial, divide each term of the polynomial by the monomial:
$$\begin{aligned}\frac{21x^2 - 14x}{7x} &= \frac{21x^2}{7x} - \frac{14x}{7x}\\ &= 3x - 2\end{aligned}$$

HW Assignment
Review pg. 259 # 2 – 4, 6 – 8, 10 – 12, 14 – 29