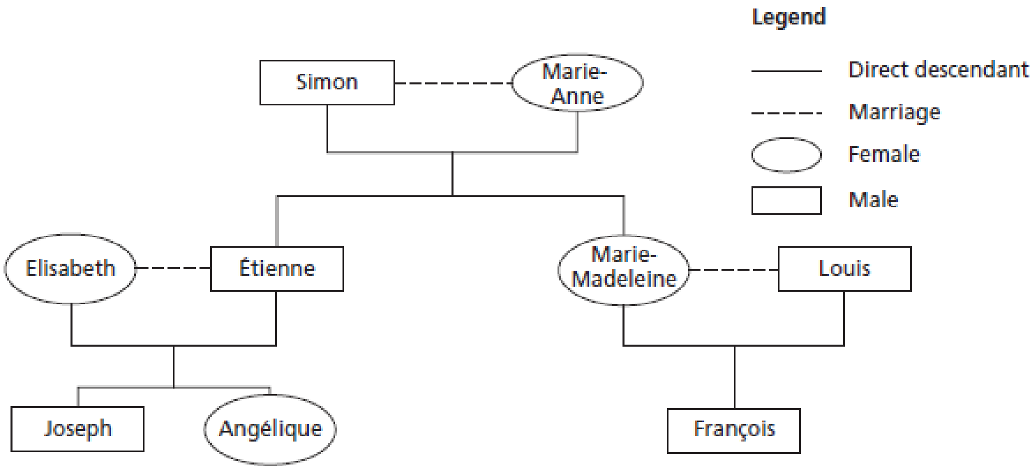
# 5.1 Representing Relations

**LESSON FOCUS:** Represent relations in different ways.

## Make Connections

This family tree shows relations within a family.



How is Joseph related to Simon?

How are Angélique and François related?

How does the family tree show these relations?

- A *set* is a collection of distinct objects.
- An *element* of a set is one object in the set.
- A **relation** associates the elements of one set with the elements of another set.

One way to write a set is to list its elements inside braces.
For example, we can write the set of natural numbers from 1 to 5 as: {1, 2, 3, 4, 5}
The order of the elements in the set does not matter.

Consider the set of fruits and the set of colours.
We can associate fruits with their colours.
For example:

| | | |
|---|---|---|
| An apple | may have the colour | red. |
| element of first set | association | element of second set |

So, this set of ordered pairs is a relation:

{(apple, red), (apple, green), (blueberry, blue), (cherry, red),
(huckleberry, blue)}

Here are some other ways to represent this relation:

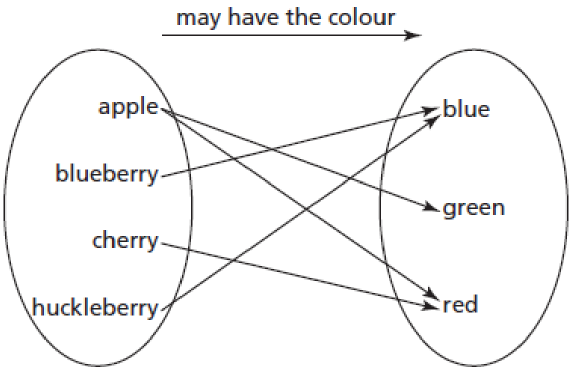
- a table

| **Fruit** | **Colour** |
|---|---|
| apple | red |
| apple | green |
| blueberry | blue |
| cherry | red |
| huckleberry | blue |

The heading of each column describes each set.

- an **arrow diagram**

The two ovals represent the sets.
Each arrow associates an element of the first set with an element of the second set.



The order of the words in the ordered pairs, the columns in the table, and the ovals in the arrow diagram is important. It makes sense to say, "an apple may have the colour red," but it makes no sense to say, "red may have the colour apple." That is, a relation has direction from one set to the other set.

## Example 1: Representing a Relation Given as a Table

Northern communities can be associated with the territories they are in. Consider the relation represented by this table.

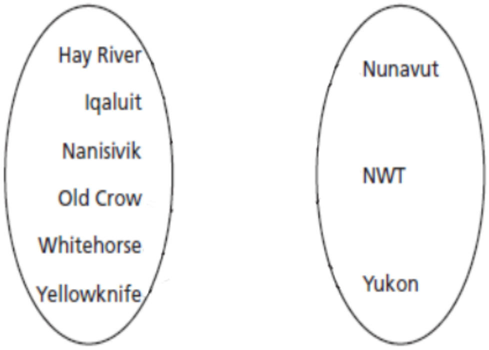
| Community | Territory |
|---|---|
| Hay River | NWT |
| Iqaluit | Nunavut |
| Nanisivik | Nunavut |
| Old Crow | Yukon |
| Whitehorse | Yukon |
| Yellowknife | NWT |

**a)** Describe this relation in words.

**b)** Represent this relation:

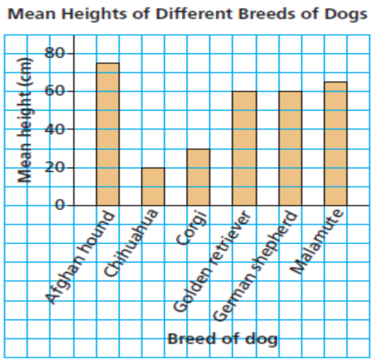
**i)** as a set of ordered pairs

**ii)** as an arrow diagram



## Example 2: Representing a Relation Given as a Bar Graph

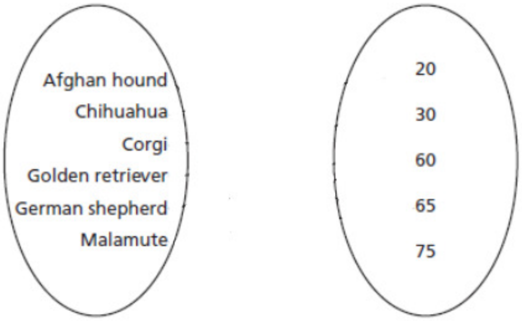
Different breeds of dogs can be associated with their mean heights. Consider the relation represented by this graph. Represent the relation:



**a)** as a table

**b)** as an arrow diagram

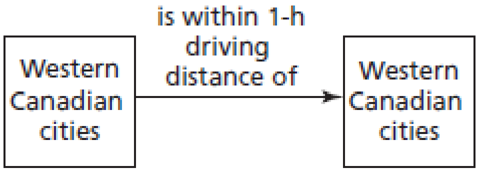
| | |
|---|---|
| | |
| | |
| | |
| | |
| | |
| | |



Sometimes a relation contains so many ordered pairs that it is impossible to list all of them or to represent them in a table.

**Example 3: Identifying a Relation from a Diagram**

In this diagram:



Both sets in a relation can be the same.

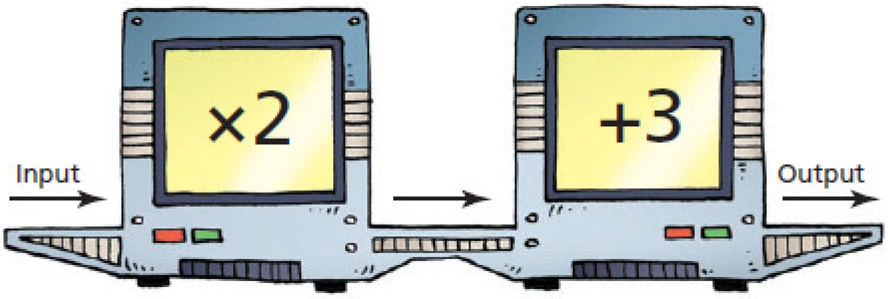
**a)** Describe the relation in words.

**b)** List 2 ordered pairs that belong to the relation.

**Homework: Page 262 #3, 4, 6 – 10**

# 5.2 Properties of Functions

**LESSON FOCUS:** Develop the concept of a function.



## Make Connections

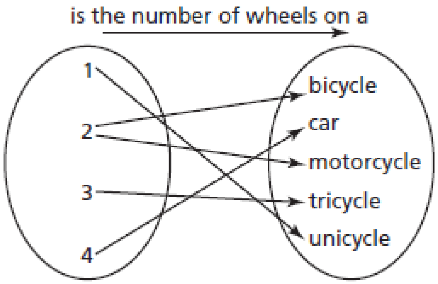
What is the rule for the Input/Output machine above?
Which numbers would complete this table for the machine?

| Input | Output |
|---|---|
| 1 | 5 |
| 2 | 7 |
|  | 9 |
| 4 |  |
|  | 13 |

- The set of first elements of a relation is called the **domain.**
- The set of related second elements of a relation is called the **range.**
- A **function** is a special type of relation where each element in the domain is associated with exactly one element in the range.

Here are some different ways to relate vehicles and the number of wheels each has.

This relation associates a number with a vehicle with that number of wheels.

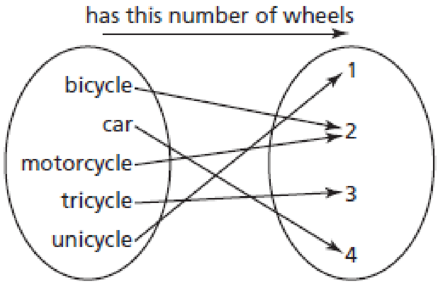


This diagram does not represent a function because there is one element in the first set that associates with two elements in the second set; that is, there are two arrows from 2 in the first set.

{(1, unicycle), (2, bicycle),
(2, motorcycle), (3, tricycle), (4, car)}

The set of ordered pairs above does not represent a function because two ordered pairs have the same first element.
The domain is the set of first elements: {1, 2, 3, 4}
The range is the set of associated second elements: {unicycle, bicycle, motorcycle, tricycle, car}

This relation associates a vehicle with the number of wheels it has.



This diagram represents a function because each element in the first set associates with exactly one element in the second set; that is, there is only one arrow from each element in the first set.

{(unicycle, 1), (bicycle, 2),
(motorcycle, 2), (tricycle, 3), (car, 4)}

The set of ordered pairs above represents a function because the ordered pairs have different first elements.
The domain is the set of first elements: {unicycle, bicycle, motorcycle, tricycle, car}
The range is the set of associated second elements: {1, 2, 3, 4}

When we list the elements of the range, we do not repeat an element that occurs more than once.

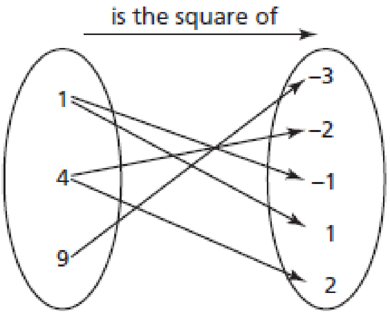
## Example 1: Identifying Functions

For each relation below:

- Determine whether the relation is a function. Justify the answer.
- Identify the domain and range of each relation that is a function.

**a)** A relation that associates given shapes with the number of right angles in the shape:
{(right triangle, 1), (acute triangle, 0), (square, 4), (rectangle, 4), (regular hexagon, 0)}

**b)**



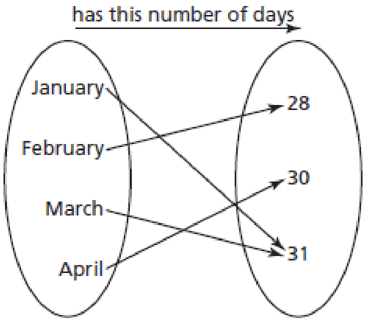
## CHECK YOUR UNDERSTANDING

For each relation below:

- Determine whether the relation is a function. Justify your answer.
- Identify the domain and range of each relation that is a function.

**a)** A relation that associates a number with a prime factor of the number:
{(4, 2), (6, 2), (6, 3), (8, 2), (9, 3)}

**b)**



[Answers: a) no b) yes; domain: {January, February,March, April}; range: {28, 30, 31}]

In the workplace, a person's gross pay, $P$ dollars, often depends on the number of hours worked, $h$.
So, we say $P$ is the *dependent variable*. Since the number of hours worked, $h$, does not depend on the gross pay, $P$, we say that $h$ is the *independent variable*.

independent variable →

← dependent variable

| Hours Worked, $h$ | Gross Pay, $P$ ($) |
|---|---|
| 1 | 12 |
| 2 | 24 |
| 3 | 36 |
| 4 | 48 |
| 5 | 60 |

domain

range

A table of values usually represents a sample of the ordered pairs in a relation.

The values of the independent variable are listed in the first column of a table of values. These elements belong to the domain.

The values of the dependent variable are listed in the second column of a table of values. These elements belong to the range.

## Example 2: Describing Functions

The table shows the masses, $m$ grams, of different numbers of identical marbles, $n$.

| Number of Marbles, $n$ | Mass of Marbles, $m$ (g) |
|---|---|
| 1 | 1.27 |
| 2 | 2.54 |
| 3 | 3.81 |
| 4 | 5.08 |
| 5 | 6.35 |
| 6 | 7.62 |

**a)** Why is this relation also a function?

**b)** Identify the independent variable and the dependent variable. Justify the choices.

**c)** Write the domain and range.

## CHECK YOUR UNDERSTANDING

The table shows the costs of student bus tickets, $C$ dollars, for different numbers of tickets, $n$.

| Number of Tickets, $n$ | Cost, $C$ ($) |
|---|---|
| 1 | 1.75 |
| 2 | 3.50 |
| 3 | 5.25 |
| 4 | 7.00 |
| 5 | 8.75 |

**a)** Why is this relation also a function?

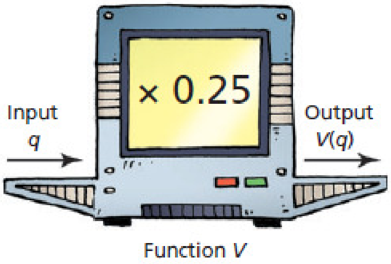
**b)** Identify the independent variable and the dependent variable. Justify the choices.

**c)** Write the domain and range.

[Answers: b) $n$; $C$ c) {1, 2, 3, 4, 5,…}; {1.75, 3.50, 5.25, 7.00, 8.75,…}]

We can think of a function as an input/output machine. The input can be any number in the domain, and the output depends on the input number. So, the input is the independent variable and the output is the dependent variable. Consider two machines that both accept quarters. Machine A calculates the value of the quarters. Machine B weighs the quarters. Each machine performs a different operation, so the machines represent two different functions.

### Machine A



When the input is $q$ quarters, the output or value, $V$, in dollars is: $0.25q$.
The equation $V = 0.25q$ describes this function.

Since $V$ is a function of $q$, we can write this equation using **function notation**:
$V(q) = 0.25q$
We say: "$V$ of $q$ is equal to $0.25q$."

This notation shows that $V$ is the dependent variable and that $V$ depends on $q$.

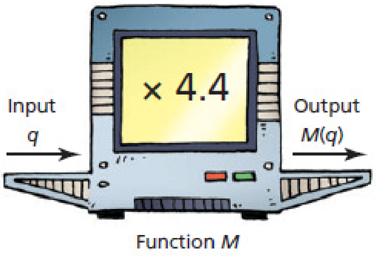
$V(3)$ represents the value of the function when $q = 3$.

$V(3) = 0.25(3)$

$V(3) = 0.75$

So, the value of 3 quarters is $0.75.

### Machine B



The mass of 1 quarter is 4.4 g.

When the input is $q$ quarters, the output or mass, $M$, in grams is: $4.4q$.
The equation $M$ _ $4.4q$ describes this function.

Since $M$ is a function of $q$, we can write this equation using **function notation**:
$M(q) = 4.4q$

This notation shows that $M$ is the dependent variable and that $M$ depends on $q$.

Any function that can be written as an equation in two variables can be written in **function notation**. For example, to write the equation $d = 4t - 5$ in function notation, we may write $d(t) = 4t - 5$. $t$ represents an element of the domain and $d(t)$ represents an element of the range.

When we write an equation that is not related to a context, we use $x$ as the independent variable and $y$ as the dependent variable. Then an equation in two variables such as $y = 3x - 2$ may be written as $f(x) = 3x - 2$.

Conversely, we may write an equation in function notation as an equation in two variables. For example, for the equation $C(n) = 300 + 25n$, we write $C = 300 + 25n$.
And, for the equation $g(x) = -2x + 5$, we write $y = -2x + 5$.

**Example 3: Using Function Notation to Determine Values**
The equation $V = -0.08d + 50$ represents the volume, $V$ litres, of gas remaining in a vehicle's tank after travelling $d$ kilometres. The gas tank is not refilled until it is empty.

**a)** Describe the function. Write the equation in function notation.

**b)** Determine the value of $V(600)$. What does this number represent?

**c)** Determine the value of $d$ when $V(d) = 26$. What does this number represent?

**CHECK YOUR UNDERSTANDING**
The equation $C = 25n + 1000$ represents the cost, $C$ dollars, for a feast following an Arctic sports competition, where $n$ is the number of people attending.

**a)** Describe the function. Write the equation in function notation.

**b)** Determine the value of $C(100)$. What does this number represent?

**c)** Determine the value of $n$ when $C(n) = 5000$. What does this number represent?

[Answers: a) $C(n) = 25n + 1000$ b) $3500 c) 160]

**Homework: Page 270 #4, 5, 8, 9, 12–15, 18, 19**

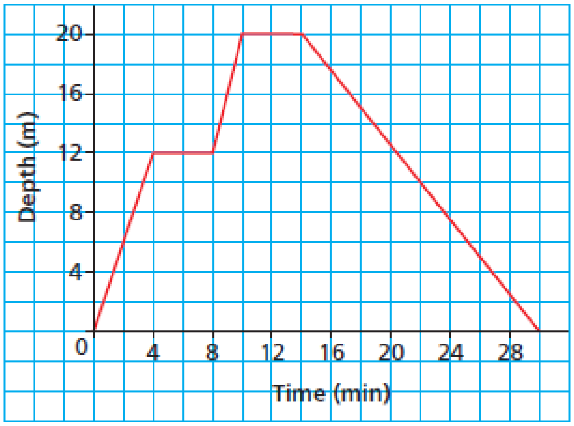
# 5.3 Interpreting & Sketching Graphs

**LESSON FOCUS:** Describe a possible situation for a given graph and sketch a possible graph for a given situation.

## Make Connections

In math, a graph provides much information.
This graph shows the depth of a scuba diver as a function of time.

A Scuba Diver's Dive

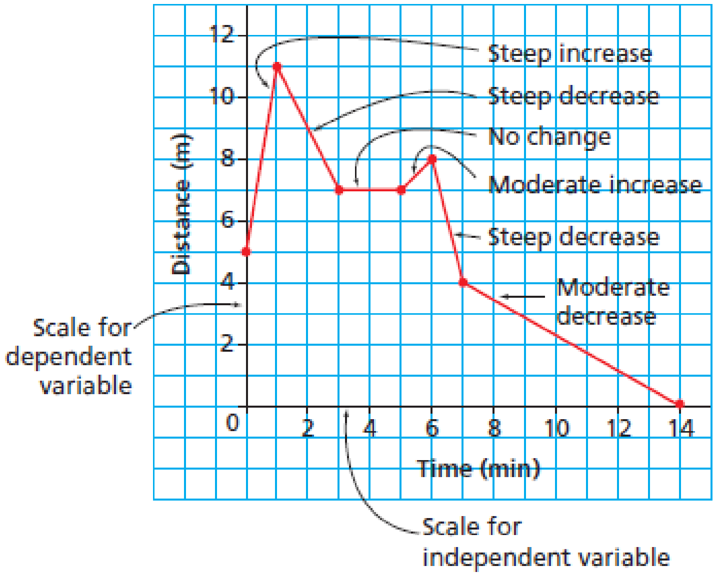


How many minutes did the dive last?

At what times did the diver stop her descent?

What was the greatest depth the diver reached? For how many minutes was the diver at that depth?

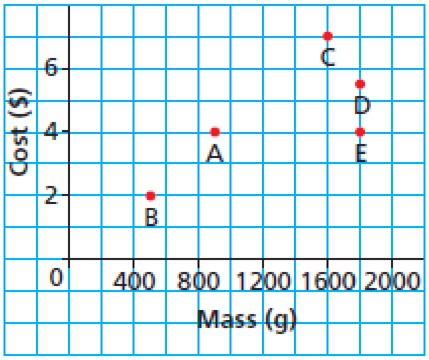
The properties of a graph can provide information about a given situation.



**Example 1: Interpreting a Graph**

Each point on this graph represents a bag of popping corn. Explain the answer to each question below.

Costs and Masses of Various Bags of Popcorn



**a)** Which bag is the most expensive? What does it cost?

**b)** Which bag has the least mass? What is this mass?

**c)** Which bags have the same mass? What is this mass?

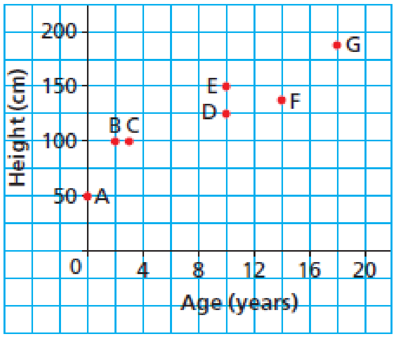
**d)** Which bags cost the same? What is this cost?

**e)** Which of bags C or D has the better value for money?

## CHECK YOUR UNDERSTANDING

Each point on this graph represents a person.
Explain your answer to each question below.

Ages and Heights of People



**a)** Which person is the oldest? What is her or his age?

**b)** Which person is the youngest? What is her or his age?

**c)** Which two people have the same height? What is this height?

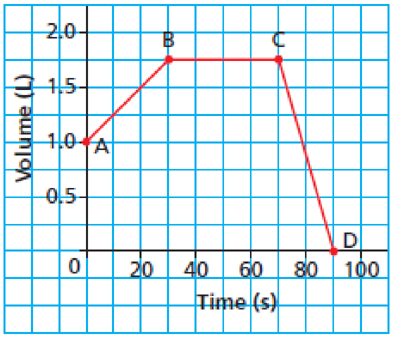
**d)** Which two people have the same age? What is this age?

**e)** Which of person B or C is taller for her or his age?

[Answers: a) G, 18 years b) A, newborn c) B and C, 100 cm d) D and E, 10 years e) B]

The graph shows how the volume of water in a watering can changes over time.
The starting volume is 1 L, which is the volume at point A.
Segment AB goes up to the right, so the volume of water is increasing from 0 s to 30 s.
Segment BC is horizontal, so the volume is constant from 30 s to 70 s.
Segment CD goes down to the right, so the volume is decreasing from 70 s to 90 s.
At point D, the volume is 0 L after 90 s.

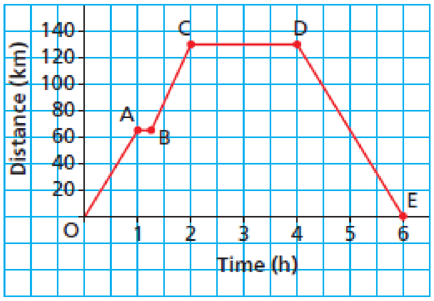
Volume of Water in a Watering Can



## Example 2: Describing a Possible Situation for a Graph

Describe the journey for each segment of the graph.

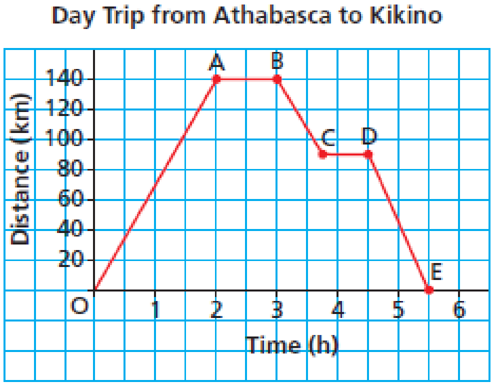
Day Trip from Winnipeg to Winkler, Manitoba



The distance between Winnipeg and Winkler is 130 km.

## CHECK YOUR UNDERSTANDING

This graph represents a day trip from Athabasca to Kikino in Alberta, a distance of approximately 140 km. Describe the journey for each segment of the graph.



[Answer: The car takes 2 h to travel 140 km to Kikino; the car stops for 1 h: the car takes approximately 45 min to travel 50 km toward Athabasca; the car stops for approximately 45 min; the car takes 1 h to travel approximately 90 km to Athabasca]

## Example 3: Sketching a Graph for a Given Situation

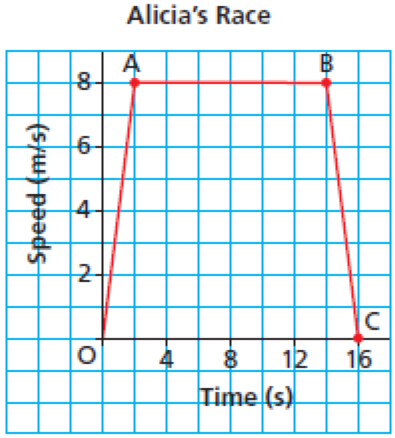
Samuel went on a bicycle ride. He accelerated until he reached a speed of 20 km/h, then he cycled for 30 min at approximately 20 km/h. Samuel arrived at the bottom of a hill, and his speed decreased to approximately 5 km/h for 10 min as he cycled up the hill. He stopped at the top of the hill for 10 min.

Sketch a graph of speed as a function of time. Label each section of the graph, and explain what it represents.

## CHECK YOUR UNDERSTANDING

At the beginning of a race, Alicia took 2 s to reach a speed of 8 m/s. She ran at approximately 8 m/s for 12 s, then slowed down to a stop in 2 s. Sketch a graph of speed as a function of time. Label each section of your graph, and explain what it represents.

Answer:



Alicia's Race

**Homework: Page 281 #3–7, 9–11, 13**

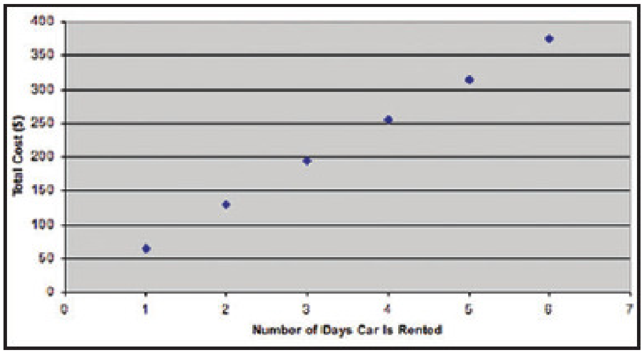
# 5.4 Math Lab: Graphing Data

**LESSON FOCUS:** Graph data and investigate the domain and range when the data represent a function.

## Make Connections

To rent a car for less than one week from Ace Car Rentals, the cost is $65 per day for the first three days, then $60 a day for each additional day.

| Number of Days Car Is Rented | Total Cost ($) |
|---|---|
| 1 | 65 |
| 2 | 130 |
| 3 | 195 |
| 4 | 255 |
| 5 | 315 |
| 6 | 375 |



Why are the points on the graph not joined?

Is this relation a function? How can you tell?

What is the domain? What is the range?

# Construct Understanding

## TRY THIS

Work with a partner.
You will need:
- a length of rope
- a metre stick
- grid paper

When you tie knots in a rope, the length of the rope is related to the number of knots tied.

**A. You will investigate the relation between the number of knots and the length of rope.**

- Measure the length of the rope without any knots. Tie a knot in the rope. Measure the length of the rope with the knot.
- Repeat the measurements for up to 5 knots. Try to tie the knots so they are all the same size and tightness.
- Record the number of knots and the length of the rope in a table.

| Number of Knots | Length of Rope (cm) |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |

**B. Graph the data.**

- How did you determine on which axis to plot each variable?

- How did you choose the scale for each axis so the data fit on the axes?

- Did you join the points? Justify your answer.

- Is the length of the rope a function of the number of knots? Explain.
  If your answer is yes, list the set of ordered pairs.

  What is the domain?

  What is the range?

- Would it make sense to extend the graph to the right? To the left?

  If your answer is yes, how far could you extend it?
  What is the new domain?

  What is the new range?

  If your answer is no, what restrictions are there on the domain and range?

**D. Suppose you combined your data with those of 4 pairs of classmates.**

- When you graph all the data, does the graph represent a function? Justify your answer.

- Suppose you calculated the mean rope length for each number of knots, then graphed the data.
  Would the graph represent a function? Justify your answer.

**Homework: Page 286 All**

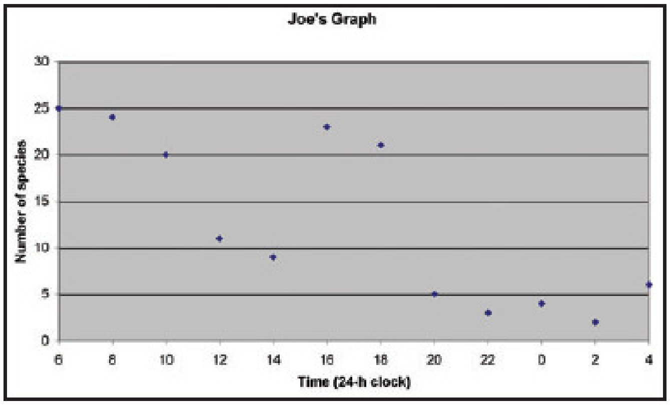
# 5.5 Graphs of Relations & Functions

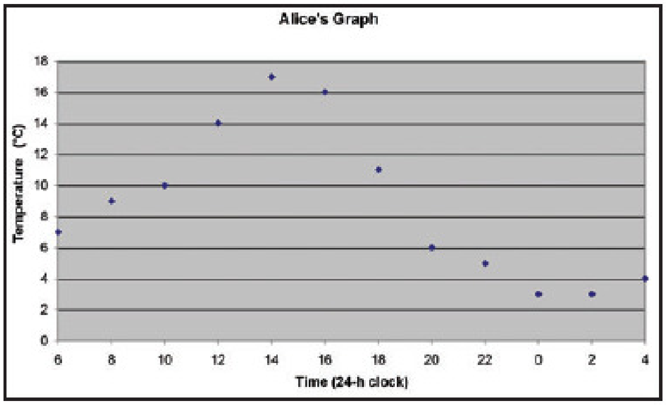
**LESSON FOCUS:** Determine the properties of the graphs of relations and functions.

*The great horned owl is Alberta's provincial bird.*

## Make Connections

In an environmental study in Northern Alberta, Joe collected data on the numbers of different species of birds he heard or saw in a 1-h period every 2 h for 24 h. Alice collected data on the temperature in the area at the end of each 1-h period. They plotted their data:





Does each graph represent a relation? A function? How can you tell?

Which of these graphs should have the data points connected? Explain.

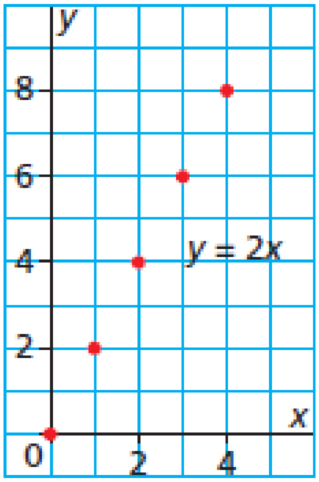
We can represent the function that associates every whole number with its double in several ways.

**Using a table of values:**

| Whole Number, $x$ | Double the Number, $y$ |
|---|---|
| 0 | 0 |
| 1 | 2 |
| 2 | 4 |
| 3 | 6 |
| 4 | 8 |

The table continues for all whole numbers. The domain is the set of whole numbers. The range is the set of even whole numbers.

**Using a graph:**



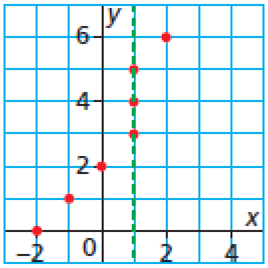
We know the relation $y = 2x$ is a function because each value of $x$ associates with exactly one value of $y$, and each ordered pair has a different first element.

The ***domain*** of a function is the set of values of the independent variable; for the graph above, the domain is the $x$-values.

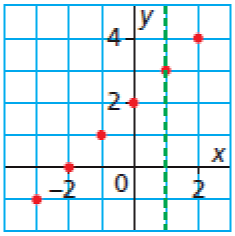
The ***range*** of a function is the set of values of the dependent variable; for the graph above, the range is the $y$-values.

When the domain is restricted to a set of discrete values, the points on the graph are not connected.

A relation that is not a function has two or more ordered pairs with the same first coordinate. So, when the ordered pairs of the relation are plotted on a grid, a vertical line can be drawn to pass through more than one point.



A function has ordered pairs with different first coordinates. So, when the ordered pairs of the function are plotted on a grid, any vertical line drawn will always pass through no more than one point.

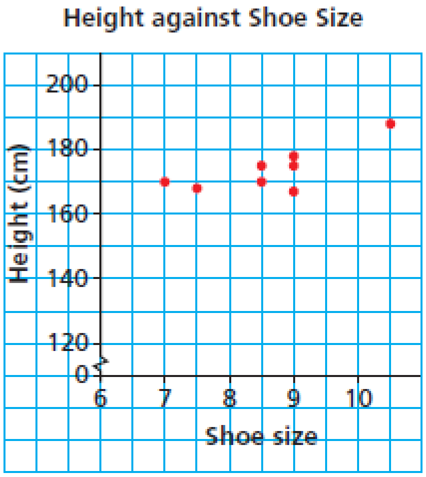


## Vertical Line Test for a Function

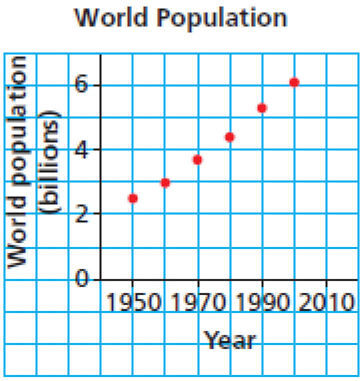
A graph represents a function when no two points on the graph lie on the same vertical line.
Place a ruler vertically on a graph, then slide the ruler across the graph.
If one edge of the ruler always intersects the graph at no more than one point, the graph represents a function.

**Example 1: Identifying whether a Graph Represents a Function**
Which of these graphs represents a function? Justify the answer.

a)

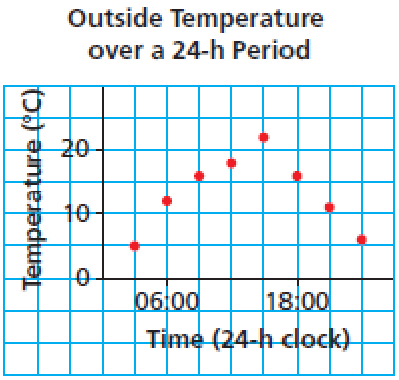


b)

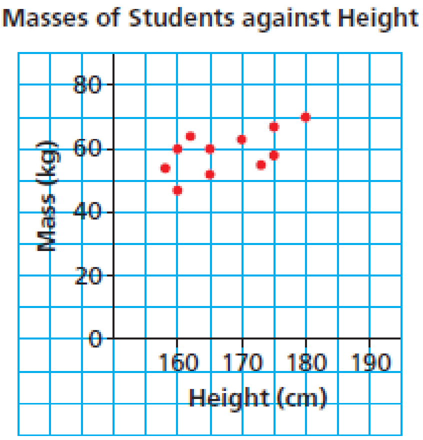


**CHECK YOUR UNDERSTANDING**
Which of these graphs represents a function? Justify your answer.

a)



b)

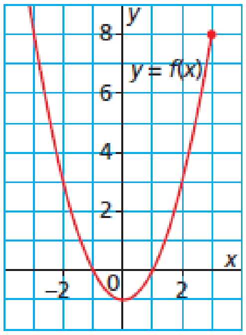


[Answers: a) function, b) not a function]

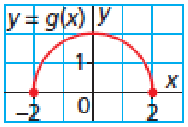
## Example 2: Determining the Domain and Range of the Graph of a Function

Determine the domain and range of the graph of each function.

a)



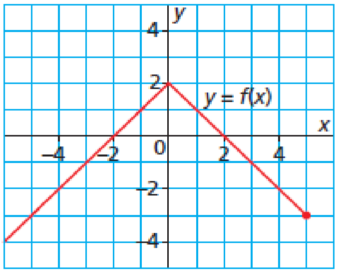
b)



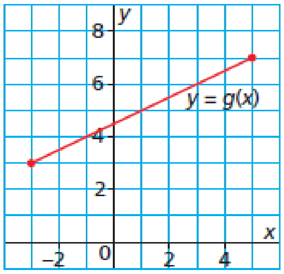
## CHECK YOUR UNDERSTANDING

Determine the domain and range of the graph of each function.

a)



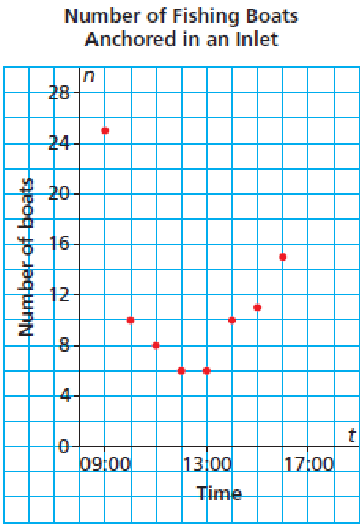
b)



[Answers: a) $x \le 5;\ y \le 2$, b) $-3 \le x \le 5;\ 3 \le y \le 7$ ]

## Example 3: Determining the Domain and Range of the Graph of a Situation

This graph shows the number of fishing boats, *n*, anchored in an inlet in the Queen Charlotte Islands as a function of time, *t*.



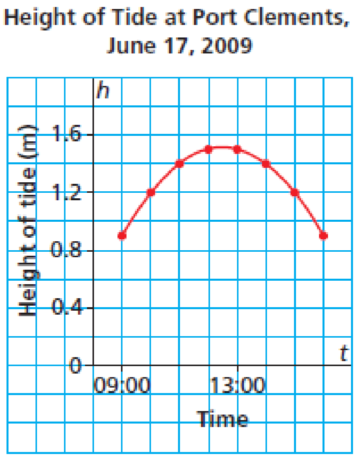
**a)** Identify the dependent variable and the independent variable.

**b)** Why are the points on the graph not connected?

**c)** Determine the domain and range of the graph.

## CHECK YOUR UNDERSTANDING

This graph shows the approximate height of the tide, *h* metres, as a function of time, *t*, at Port Clements, Haida Gwaii on June 17, 2009.



**a)** Identify the dependent variable and the independent variable.

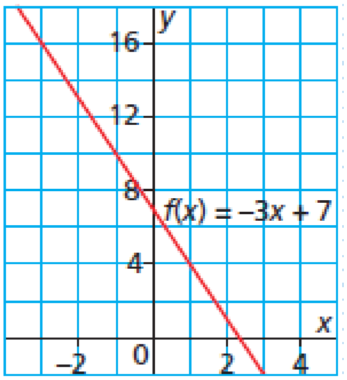
**b)** Why are the points on the graph connected?

**c)** Determine the domain and range of the graph.

[Answers: a) *h*, *t*  c) $09{:}00 \le t \le 16{:}00$;  $0.9 \le h \le 1.5$ ]

## Example 4: Determining Domain Values and Range Values from the Graph of a Function

Here is a graph of the function $f(x) = -3x + 7$

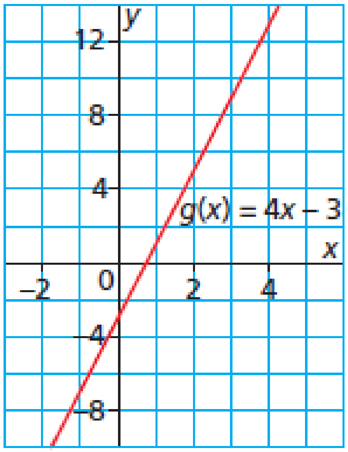


**a)** Determine the range value when the domain value is $-2$.

**b)** Determine the domain value when the range value is 4.

## CHECK YOUR UNDERSTANDING

Here is a graph of the function $g(x) = 4x - 3$.



**a)** Determine the range value when the domain value is 3.

**b)** Determine the domain value when the range value is $-7$.

Answers: a) 9 b) –1

**Homework: Page 294 #7–10, 13, 16, 17**

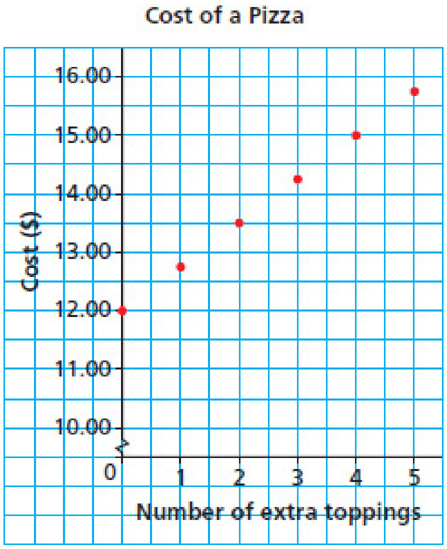
# 5.6 Properties of Linear Relations

**LESSON FOCUS:** Identify and represent linear relations in different ways.

## Make Connections

The table of values and graph show the cost of a pizza with up to 5 extra toppings.

| Number of Extra Toppings | Cost ($) |
|---|---|
| 0 | 12.00 |
| 1 | 12.75 |
| 2 | 13.50 |
| 3 | 14.25 |
| 4 | 15.00 |
| 5 | 15.75 |



What patterns do you see in the table?

Write a rule for the pattern that relates the cost of a pizza to the number of its toppings.

How are the patterns in the table shown in the graph?

How can you tell from the table that the graph represents a linear relation?

The cost for a car rental is \$60, plus \$20 for every 100 km driven.
The independent variable is the distance driven and the dependent variable is the cost.
We can identify that this is a linear relation in different ways.

- a table of values

Independent variable → ← Dependent variable

| Distance (km) | Cost (\$) |
|---|---|
| 0 | 60 |
| 100 | 80 |
| 200 | 100 |
| 300 | 120 |
| 400 | 140 |

+100 +100 +100 +100

+20 +20 +20 +20

For a linear relation, a constant change in the independent variable results in a constant change in the dependent variable.

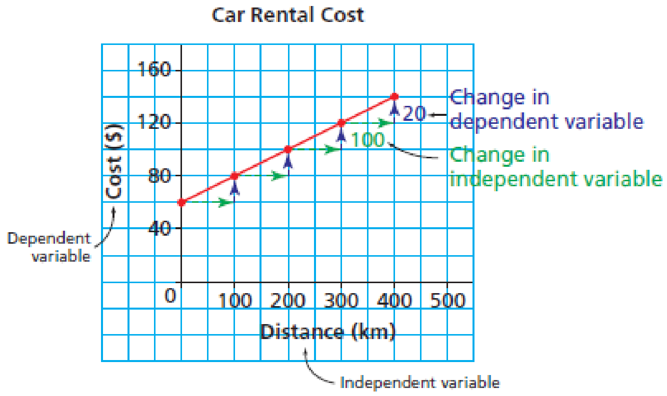
- a set of ordered pairs

+100 +100 +100 +100

$$\{(0, 60), (100, 80), (200, 100), (300, 120), (400, 140)\}$$

+20 +20 +20 +20

- a graph



We can use each representation above to calculate the **rate of change.**
The rate of change can be expressed as a fraction:

$$\frac{\text{change in dependent variable}}{\text{change in independent variable}} = \frac{\$20}{100\text{ km}}$$

$$= \$0.20/\text{km}$$

The rate of change is $0.20/km; that is, for each additional 1 km driven, the rental cost increases by 20¢. The rate of change is constant for a linear relation.

We can determine the rate of change from the equation that represents the linear function.
Let the cost be $C$ dollars and the distance driven be $d$ kilometres.
An equation for this linear function is:

$$C = 0.20d + 60$$

initial amount
independent variable
rate of change
dependent variable

## Example 1: Determining whether a Table of Values Represents a Linear Relation

Which table of values represents a linear relation? Justify the answer.

**a)** The relation between temperature in degrees Celsius, $C$, and temperature in degrees Fahrenheit, $F$

| $C$ | $F$ |
|---|---|
| 0 | 32 |
| 5 | 41 |
| 10 | 50 |
| 15 | 59 |
| 20 | 68 |

**b)** The relation between the current, $I$ amps, and power, $P$ watts, in an electrical circuit

| $I$ | $P$ |
|---|---|
| 0 | 0 |
| 5 | 75 |
| 10 | 300 |
| 15 | 675 |
| 20 | 1200 |

## CHECK YOUR UNDERSTANDING

Which table of values represents a linear relation? Justify the answer. [Answers: a) not linear b) linear]

**a)** The relation between the number of bacteria in a culture, $n$, and time, $t$ minutes.

| $t$ | $n$ |
|---|---|
| 0 | 1 |
| 20 | 2 |
| 40 | 4 |
| 60 | 8 |
| 80 | 16 |
| 100 | 32 |

**b)** The relation between the amount of goods and services tax charged, $T$ dollars, and the amount of the purchase, $A$ dollars

| $A$ | $T$ |
|---|---|
| 60 | 3 |
| 120 | 6 |
| 180 | 9 |
| 240 | 12 |
| 300 | 15 |

When an equation is written using the variables *x* and *y*,
***x* represents the independent variable** and ***y* represents the dependent variable**.

**Example 2: Determining whether an Equation Represents a Linear Relation**

**a)** Graph each equation.

i) $y = -3x + 25$

| | |
|---|---|
| *x* | *y* |
| | |
| | |
| | |
| | |
| | |
| | |
| | |

ii) $y = 2x^2 + 5$

| | |
|---|---|
| *x* | *y* |
| | |
| | |
| | |
| | |
| | |
| | |
| | |

iii) $y = 5$

| | |
|---|---|
| *x* | *y* |
| | |
| | |
| | |
| | |
| | |
| | |
| | |

iv) $x = 1$

| | |
|---|---|
| *x* | *y* |
| | |
| | |
| | |
| | |
| | |
| | |
| | |

**b)** Which equations in part a represent linear relations? How do you know?

**Example 3: Identifying a Linear Relation**

Which relation is linear? Justify the answer.

**a)** A new car is purchased for $24 000. Every year, the value of the car decreases by 15%. The value is related to time.

**b)** For a service call, an electrician charges a $75 flat rate, plus $50 for each hour he works. The total cost for service is related to time.

## CHECK YOUR UNDERSTANDING

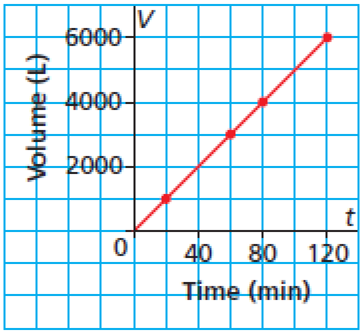
Which relation is linear? Justify your answer. [Answers: a) linear, b) not linear]

**a)** A dogsled moves at an average speed of 10 km/h along a frozen river. The distance travelled is related to time.

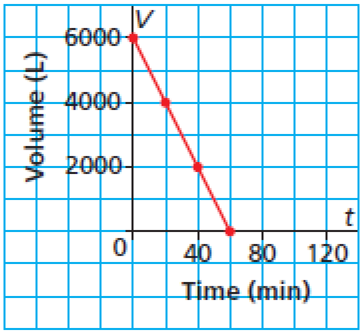
**b)** The area of a square is related to the side length of the square.

## Example 4: Determining the Rate of Change of a Linear Relation from Its Graph

A water tank on a farm near Swift Current, Saskatchewan, holds 6000 L.
Graph A represents the tank being filled at a constant rate.
Graph B represents the tank being emptied at a constant rate.



Graph A
Filling a Water Tank



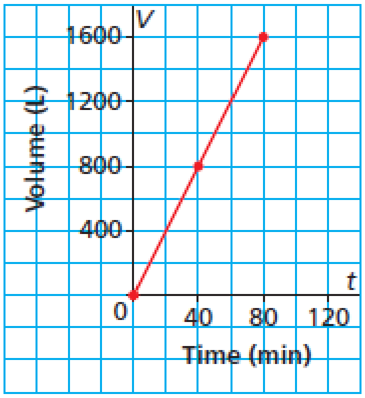
Graph B
Emptying a Water Tank

a) Identify the independent and dependent variables.

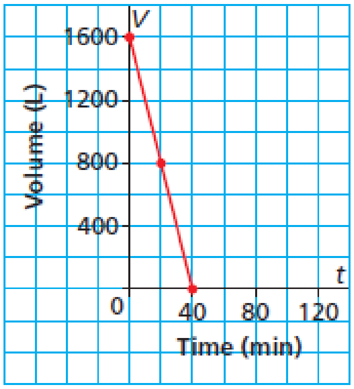
b) Determine the rate of change of each relation, then describe what it represents.

## CHECK YOUR UNDERSTANDING

A hot tub contains 1600 L of water. Graph A represents the hot tub being filled at a constant rate. Graph B represents the hot tub being emptied at a constant rate.



Graph A
Filling a Hot Tub



Graph B
Emptying a Hot Tub

a) Identify the independent and dependent variables.

b) Determine the rate of change of each relation, then describe what it represents.

[Answers: Graph A a) *V*, *t* b) 20 L/min, Graph B a) *V*, *t* b) _40 L/min]

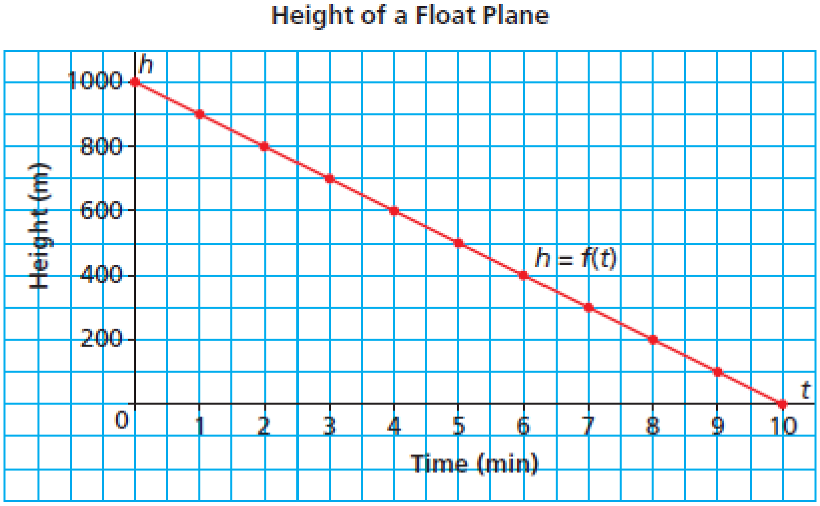
**Homework: Page 308 #3ac, 4ac, 6, 7, 9–12, 15, 16**

# 5.7 Interpreting Graphs of Linear Functions

**LESSON FOCUS:** Use intercepts, rate of change, domain, and range to describe the graph of a linear function.

## Make Connections

Float planes fly into remote lakes in Canada's Northern wilderness areas for ecotourism. This graph shows the height of a float plane above a lake as the plane descends to land.



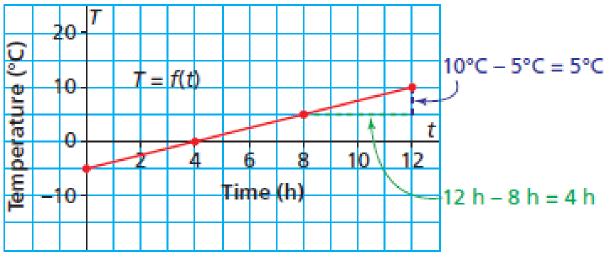
Where does the graph intersect the vertical axis? What does this point represent?

Where does the graph intersect the horizontal axis?
What does this point represent?

What is the rate of change for this graph? What does it represent?

Each graph below shows the temperature, $T$ degrees Celsius, as a function of time, $t$ hours, for two locations.

Temperature in Location A



The point where the graph intersects the horizontal axis has coordinates (4, 0). The **horizontal intercept** is 4. This point of intersection represents the time, after 4 h, when the temperature is 0°C.

The point where the graph intersects the vertical axis has coordinates (0, –5). The **vertical intercept** is –5. This point of intersection represents the initial temperature, –5°C.

The *domain* is: $0 \le t \le 12$

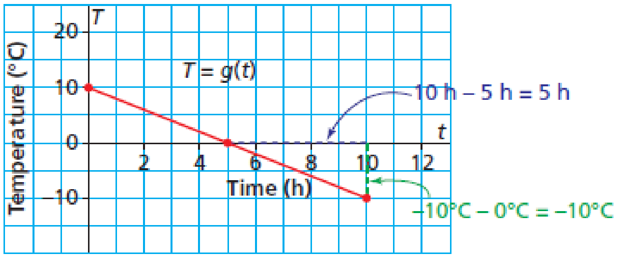
The *range* is: $-5 \le T \le 10$

The *rate of change* is: $\frac{\text{change in } T}{\text{change in } t} = \frac{5°\text{C}}{4\,\text{h}}$

$$= 1.25°\text{C/h}$$

The rate of change is positive because the temperature is increasing over time.

Temperature in Location B



The point where the graph intersects the horizontal axis has coordinates (5, 0). The *horizontal intercept* is 5. This point of intersection represents the time, after 5 h, when the temperature is 0°C.

The point where the graph intersects the vertical axis has coordinates (0, 10). The *vertical intercept* is 10. This point of intersection represents the initial temperature, 10°C.

The *domain* is: $0 \le t \le 10$

The *range* is: $-10 \le T \le 10$

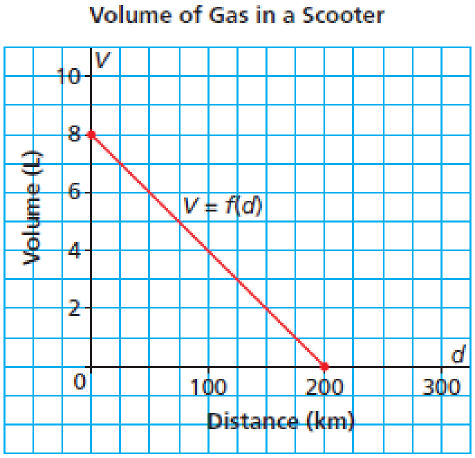
The *rate of change* is: $\frac{\text{change in } T}{\text{change in } t} = \frac{-10°\text{C}}{5\,\text{h}}$

$$= -2°\text{C/h}$$

The rate of change is negative because the temperature is decreasing over time.

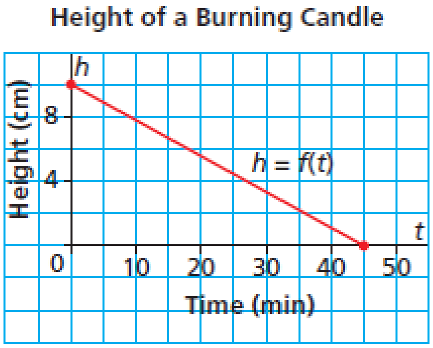
**Example 1: Determining Intercepts, Domain, and Range of the Graph of a Linear Function**
This graph shows the fuel consumption of a scooter with a full tank of gas at the beginning of a journey.

**a)** Write the coordinates of the points where the graph intersects the axes. Determine the vertical and horizontal intercepts. Describe what the points of intersection represent.



**b)** What are the domain and range of this function?

**CHECK YOUR UNDERSTANDING**
This graph shows how the height of a burning candle changes with time.



**a)** Write the coordinates of the points where the graph intersects the axes. Determine the vertical and horizontal intercepts. Describe what the points of intersection represent.

**b)** What are the domain and range of this function?

[Answers: a) (0, 10), 10; (45, 0), 45
b) domain: $0 \le t \le 45$; range:
$0 \le h \le 10$]

### Example 2: Sketching a Graph of a Linear Function in Function Notation

Sketch a graph of the linear function $f(x) = -2x + 7$.

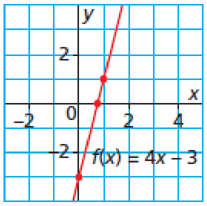
| | |
|---|---|
| $x$ | $y$ |
| | |
| | |
| | |
| | |
| | |
| | |
| | |

## CHECK YOUR UNDERSTANDING

Sketch a graph of the linear function $f(x) = 4x - 3$.

| | |
|---|---|
| $x$ | $y$ |
| | |
| | |
| | |
| | |
| | |
| | |
| | |

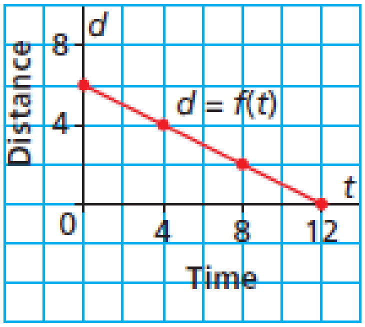
Answer:



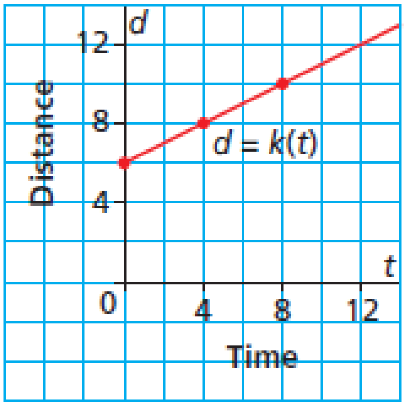
## Example 3: Matching a Graph to a Given Rate of Change and Vertical Intercept

Which graph has a rate of change of $\frac{1}{2}$ and a vertical intercept of 6? Justify the answer.

a)



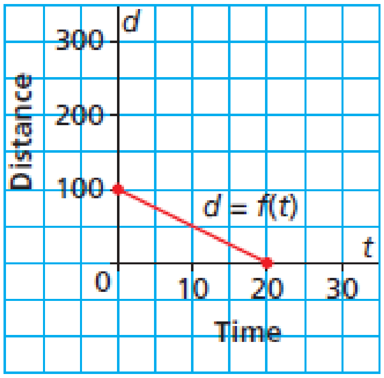
b)



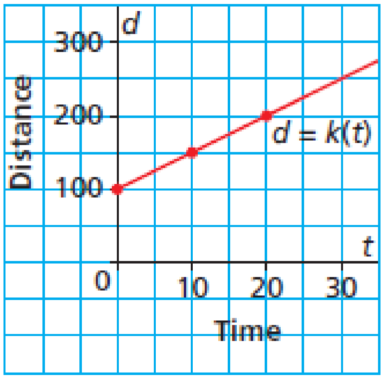
## CHECK YOUR UNDERSTANDING

Which graph has a rate of change of $-5$ and a vertical intercept of 100? Justify your answer. [Answer: the graph in part a]

a)

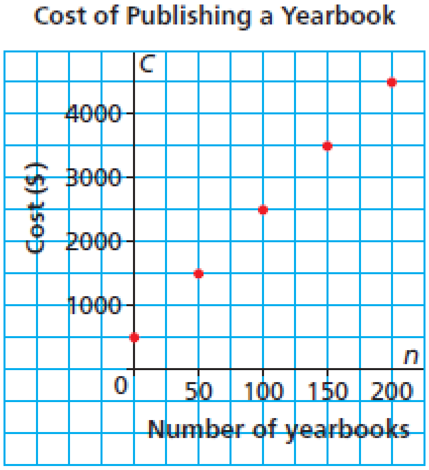


b)



## Example 4: Solving a Problem Involving a Linear Function

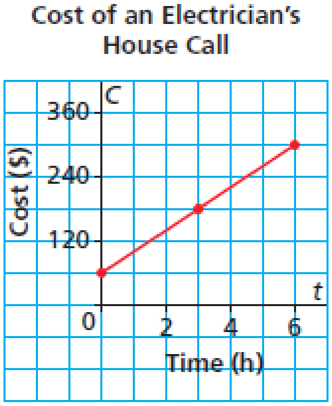
This graph shows the cost of publishing a school yearbook for Collège Louis-Riel in Winnipeg.



The budget for publishing costs is $4200.What is the maximum number of books that can be printed?

## CHECK YOUR UNDERSTANDING

This graph shows the total cost for a house call by an electrician for up to 6 h work.



The electrician charges $190 to complete a job. For how many hours did she work?

[Answer: $3\frac{1}{4}$ h]

**Homework: Page 319 #6–8, 10, 13, 16, 17**
**Chapter Review: Page 326 #1–18**