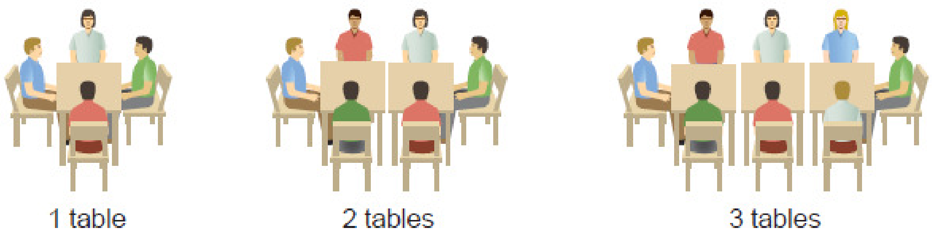
4.1 – Writing Equations to Describe Patterns

Focus: Use equations to describe and solve problems involving patterns.

Warm-up: Do the 'Investigate. Describe the pattern:

Investigate

A banquet hall has small square tables that seat 1 person on each side. The tables can be pushed together to form longer tables.



The pattern continues.

- Sketch the next 2 table arrangements in the pattern.
  What stays the same in each arrangement? What changes?
- What different strategies can you use to determine the number of people at 6 tables? At 10 tables? At 25 tables?

Describe the pattern:

a) in words.

b) in a table.

Tables	People
1	4
2	
3	

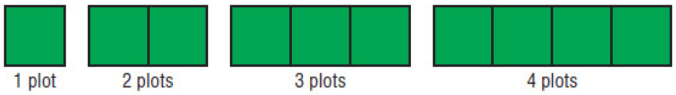
c) in an equation where t = number of tables & p = number of people.

d) How many people at 25 tables?

Together, read through 'Connect' and make any notes you think are important on the side of the page.

Connect

A landscape designer uses wooden boards as edging for the plots in a herb garden.



The number of boards, b, is *related* to the number of plots, p.
This relationship can be represented in different ways:

- using pictures
- using a table of values
- using an equation

Here are 2 ways to determine the equation.

➤ Determine a pattern in the number of boards.

Number of Plots, p	Number of Boards, b
1	4
2	7
3	10
4	13

(+1 between successive values of p; +3 between successive values of b)

As the number of plots increases by 1, the number of boards increases by 3. Repeated addition of 3 is the same as multiplication by 3. This suggests that the number of boards may be 3 times the number of plots. So, the equation $b = 3p$ may represent this relationship.

Check whether the equation $b = 3p$ is correct.
When $p = 1$,
$b = 3(1)$
$= 3$

This is 1 less than the number 4 in the table.
So, we add 1 to $3p$ to describe the number of boards correctly.

Number of Plots, p	Number of Boards, b
1	$3(1) + 1 = 4$
2	$3(2) + 1 = 7$
3	$3(3) + 1 = 10$
4	$3(4) + 1 = 13$

The terms $3p + 1$ form an *expression* that represents the number of boards for any number of plots p.
An equation is: $b = 3p + 1$

We verify the equation by substituting values of p and b from the table.
For example, check by substituting $p = 4$ and $b = 13$ in $b = 3p + 1$.
Left side: $b = 13$ Right side: $3p + 1 = 3(4) + 1$
$= 12 + 1$
$= 13$

Since the left side equals the right side, the equation is verified.

➤ Determine a pattern in the figures that represent the garden.

Number of Plots, p	Pattern in the Number of Boards	Number of Boards, b
1	$1 + 3$	$1 + 1(3)$
2	$1 + 3 + 3$	$1 + 2(3)$
3	$1 + 3 + 3 + 3$	$1 + 3(3)$
4	$1 + 3 + 3 + 3 + 3$	$1 + 4(3)$
:		:
p		$1 + p(3)$

Each garden needs 1 board for the left border and 3 additional boards for each plot.
That is,
Number of boards = 1 + (Number of plots) × 3
As an equation:
$b = 1 + p(3)$
This can be rewritten as:
$b = 1 + 3p$

Addition is commutative, so $1 + 3p = 3p + 1$.
The equation gives a general pattern rule. We say the equation *generalizes* the pattern. We can use the equation to determine the value of any term.

Ex. 1: You go to the store to buy peanut butter.

a) Make notes on or around the table to describe any patterns that you see.

Jars	**Cost ($)**
0	0
1	5
2	10
3	15
4	20

b) Write an equation that relates the cost (C) to the number of jars of peanut butter (n).

c) What is the cost of 18 jars?

Ex. 2:

a) Find the equation using the pattern in the table.

m	***n***
0	1
1	3
2	5
3	7

b) If $m = 8$, what is n?

Ex. 3:

a) Find the equation using the pattern in the table.

a	***b***
1	7
2	4
3	1
4	–2

b) If $a = 10$, what is b?

Ex. 4: Empress Cabs charges \$3.75 plus \$1.25 per km.

a) If d = number of km, write an equation for the cost of Empress Cabs.

b) What is the fare for a 19 km ride?

c) If you pay \$12.50 for your cab ride, how far did you travel?

HW Assignment
Section 4.1 pg. 159 # 4–8, 10, 11, 14–16

4.2 – Linear Relations

Focus: Analyze the graph of a linear relation.

Warm-up: Plotting points on a coordinate plane.

The ***x*–axis** is the ________________________ axis and the ***y*–axis** is ________________________ **axis.**

In any ordered pair, such as (3, – 5), the first number is **always** the *x* value and the second is the *y* value.

To plot a point on the graph, start at the origin (0, 0).

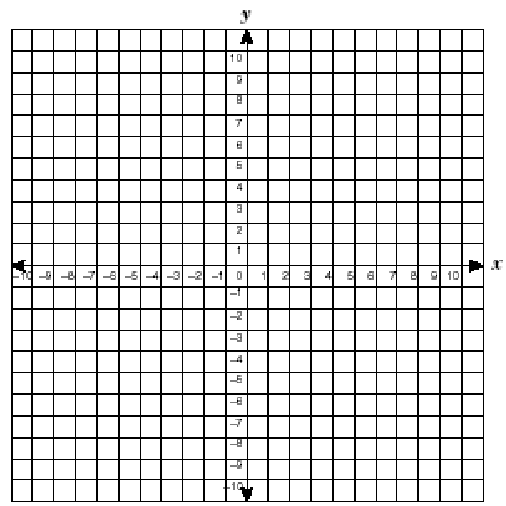
If *x* is **positive**, go _________________ and if *x* is **negative**, go ___________________.

From there, count your *y*. If *y* is **positive**, go ___________ and if *y* is negative, go ___________ .

Then plot your point.

Plot and label the points on the coordinate plane.

A (0, 0) E (5, 0)
B (0, 3) F (-4, 0)
C (3, 7) G (-8, 5)
D (0, -6) H (-4, -7)

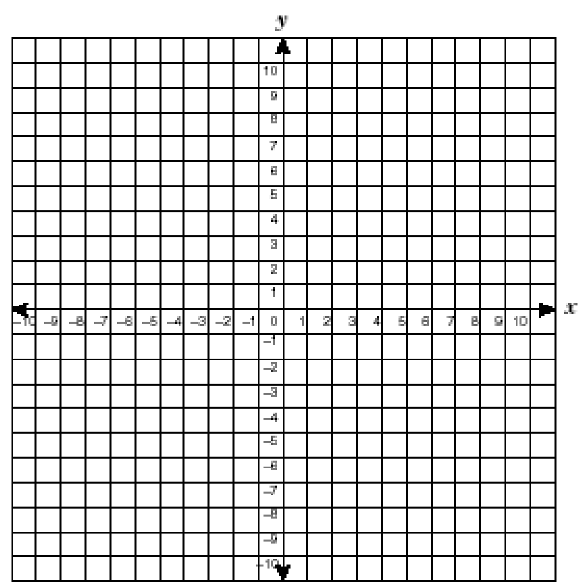


Ex. 1: Suppose you were monitoring daily temperature. Three days ago, the temperature was -7°C. Everyday since, the temperature has/will increase by 3°C.

a) Complete the table.

Day (*x*-axis)	Temp (*y*)
-3	-7
-2	
-1	
0	
1	
2	
3	

b) Graph the relation.

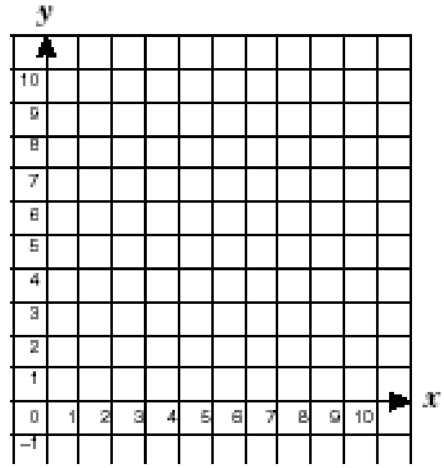


c) What kind of pattern and/or relationships do you notice in the table and/or graph?

Ex. 2: The table shows the cost of renting DVDs at an online store.

a) Graph the points, but don't draw a line.

DVDs Rented	Cost ($)
1	2.50
2	5.00
3	7.50
4	10.00



b) Use the table to describe the pattern in the rental costs. How is this pattern shown in the graph?

c) Why don't we draw a line?

Investigate:

Is the number of DVDs purchased related to the cost?

What is the equation for DVDs and cost in example 2? Use x for DVDs and y for cost.

What does the equation tell us?

Does cost depend on the number of DVDs, or does the number of DVDs depend on cost?

On what axes do we plot the two variables?

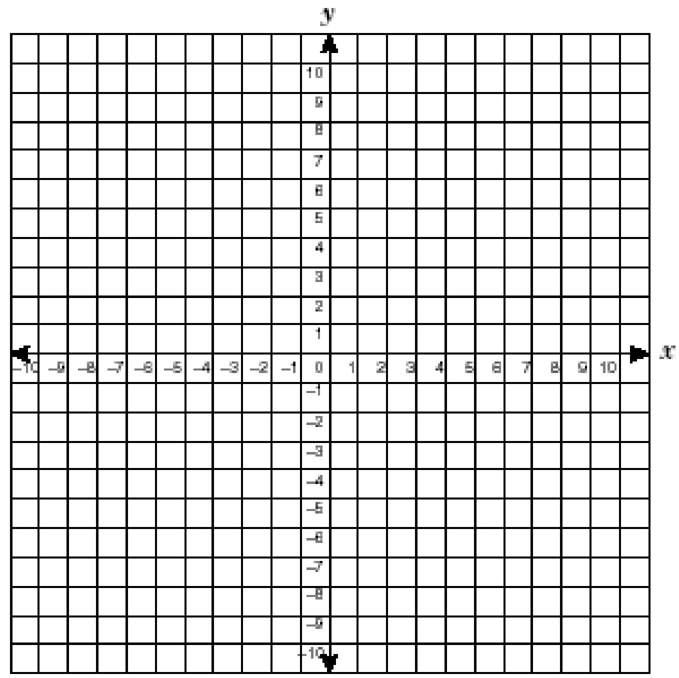
Linear Relation

When the graph of a relation is a *straight line,* we have a **linear relation.** In a linear relation, a constant change in one quantity produces a constant change in the related quantity.

Ex. 3: A relation has the equation $y = 5 - 2x$

a) Create a table of values for *x* from -2 to 4. Find the *y* value for each.

x	*y*
-2	



b) Graph the relation.
Should you join the points with a line?

c) What patterns do you see in the table and graph?

d) Is the relation linear? Explain why.

HW Assignment
Section 4.2 pg. 170 # 4, 5, 7, 10acef, 11–14

4.3 – Another Form of the Equation for a Linear Relation

Focus: Recognize the equations of horizontal, vertical, and oblique lines, and graph them.

Warm-up: Suppose you have a piece of licorice 10 cm long.

a) How many different ways could you cut it into two pieces?

b) In words, how are the lengths of the two pieces related?

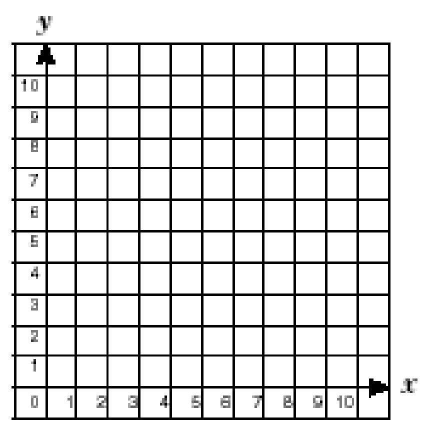
c) If x = the length of the first piece, and y = the length of the second piece, write an equation for the relation.

d) How is your equation different from the equations we worked with in 4.2?

e) Make a table of values.

Piece #1 (x)	Piece #2 (y)

f) Graph the equation.



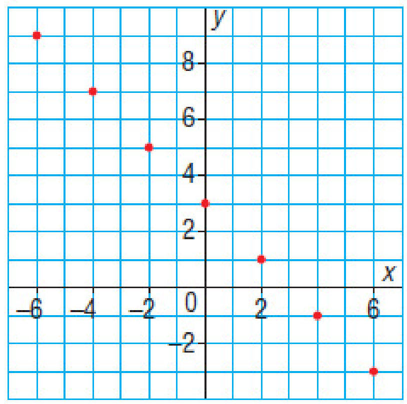
g) Is the relation linear? Why or why not?

Let's look at 'Connect'

Connect

Two integers have a sum of 3.
Let x and y represent the two integers.
Here is a table of values and a graph to represent the relation.

First Integer, x	Second Integer, y
−6	9
−4	7
−2	5
0	3
2	1
4	−1
6	−3



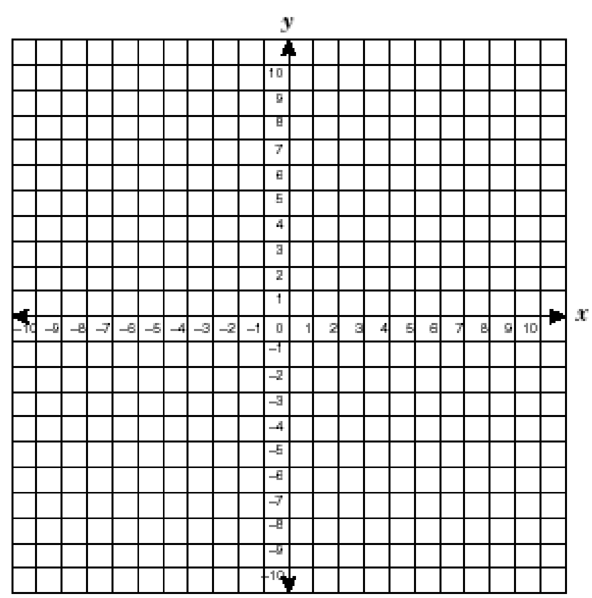
The points lie on a straight line, so the relation is linear.
We can write this linear relation as:
First integer + second integer = 3
Then, the linear relation is: $x + y = 3$
This equation has both variables on the left side of the equation.
It illustrates another way to write the equation of a linear relation.

Ex. 1: For the equation $3x - 2y = 6$:

a) Make a table of values for x = -4, 0, 4

x	y
-4	
0	
4	

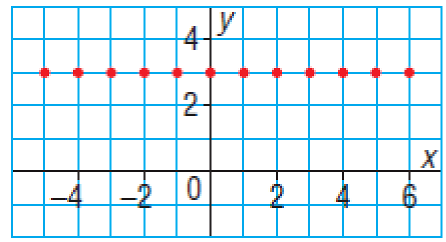
b) Graph the equation.



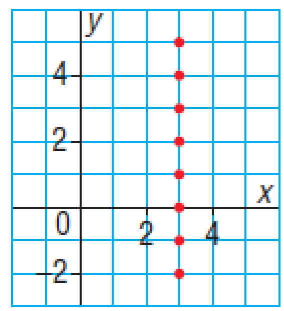
c) What is another name for a 'slanted' line?

Suppose one variable does not appear in the equation.

➤ Suppose x does not appear in $x + y = 3$.
Then we have the equation $y = 3$.
To graph this equation on a grid,
plot points that have a y-coordinate of 3.
All the points lie on a horizontal line
that is 3 units above the x-axis.

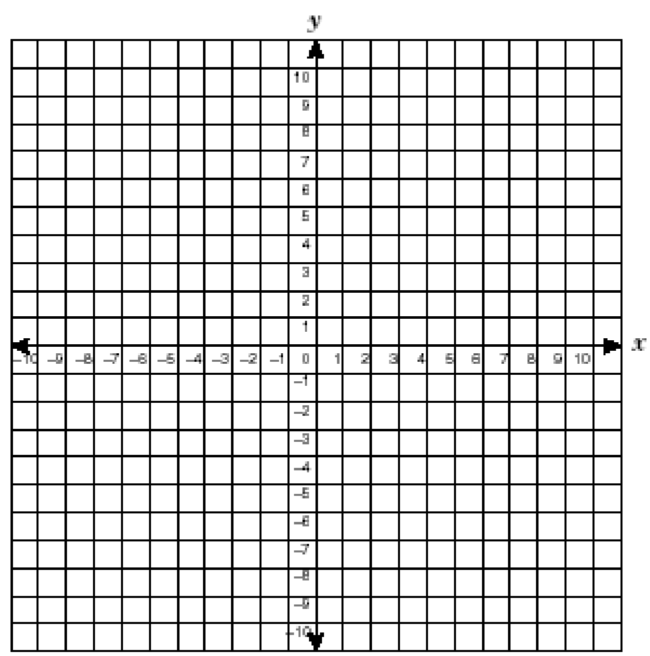


➤ Suppose y does not appear in $x + y = 3$.
Then we have the equation $x = 3$.
To graph this equation on a grid,
plot points that have an x-coordinate of 3.
All the points lie on a vertical line that is
3 units to the right of the y-axis.



Ex. 2: Graph $x = 2$

Hint: The only requirement for a point on the graph is that the x value must be 2. So y can be anything, as long as x is 2.

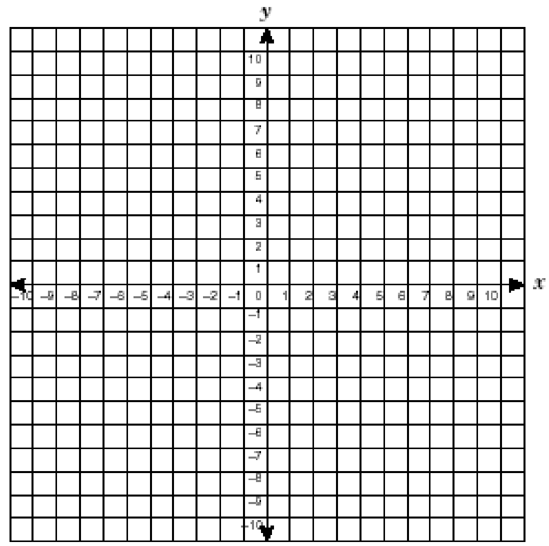


What kind of line is produced?

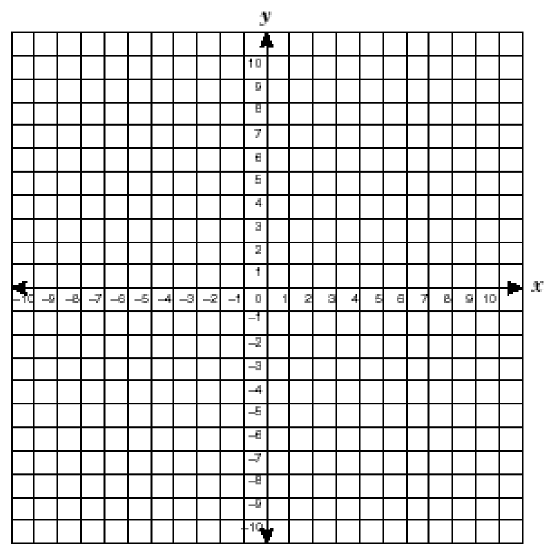
Ex. 3: Graph $y = -5$

The only requirement is that for each point, the y value must be -5.

What kind of line is produced?



Ex. 4: Graph $x = -1$ and $y = 8$



What kind of equation produces a(n):

a) oblique line?

b) vertical line?

c) horizontal line?

Reflection: Give an example of an equation that produces a:

(*i*) oblique line

(*ii*) vertical line

(*iii*) horizontal line

Quiz Next Class on 4.1 to 4.3

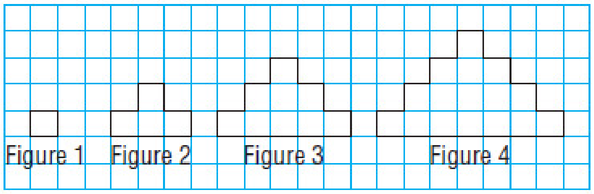
HW Assignment

Section 4.3 pg. 178 # 4–9, 12, 15, 17, 18

Mid-Unit Review

4.1

1. This pattern of squares continues.



a) Make a table that shows the figure number, n, and the perimeter of a figure, P. What patterns do you see?

b) Write an expression for the perimeter of figure n.

c) What is the perimeter of figure 40?

d) Write an equation that relates P to n.

e) Which figure has a perimeter of 136 units? How do you know?

2. A phone company charges a fixed cost of \$10 per month, plus \$0.25 per minute for long distance calling.

a) Write an equation that relates the monthly cost, C dollars, to t, the time in minutes.

b) In one month, the time for the long distance calls was 55 minutes. What was the monthly cost?

c) For one month, the cost was \$22.50. How many minutes of long distance calls were made?

4.2

3. Create a table of values for each linear relation, then graph the relation. Use values of x from -3 to 3.

a) $y = -3x$

b) $y = 2x$

c) $y = 2 - 4x$

d) $y = -2x + 4$

e) $y = -3 + x$

f) $y = -x + 3$

4. Alicia buys a \$300-jacket on lay away. She made a down payment of \$30 and is paying \$15 per week. The total paid, P dollars, after n weeks can be represented by the equation $P = 15n + 30$.

a) Create a table of values to show the total paid in each of the first 5 weeks.

b) Graph the data. Should you join the points on the graph? Explain.

c) How do the patterns in the graph relate to the patterns in the table?

5. Each table of values represents a linear relation. Copy and complete each table. Explain your reasoning.

a)

x	y
1	10
2	14
3	
4	
5	

b)

x	y
1	
3	−10
5	−14
7	
9	

c)

x	y
−2	
−1	
0	−3
1	3
2	

d)

x	y
2	
4	−2
6	−5
8	
10	

4.3

6. a) Graph each equation.

i) $y = 1$

ii) $x = -4$

iii) $x + y = 8$

iv) $2x - y = 12$

b) For which equations in part a did you *not* need to make a table of values? Explain why.

7. The difference of two numbers is 1. Let g represent the greater number and n the lesser number.

a) Complete a table for 4 different values of n.

b) Graph the data. Should you join the points? Explain.

c) Write an equation that relates n and g.

Mid-Unit Review Answers

1. **a)**

Figure Number, n	Perimeter, P
1	4
2	10
3	16
4	22

b) $6n - 2$ **c)** 238 units

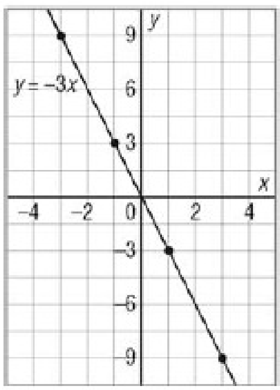
d) $P = 6n - 2$ **e)** Figure 23

2. **a)** $C = 10 + 0.25t$ **b)** $23.75

c) 50 min

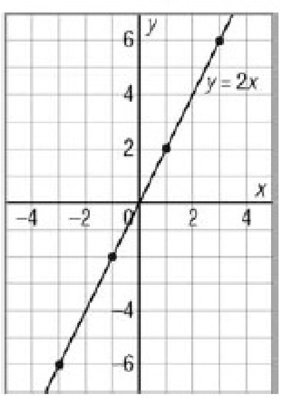
3. **a)** $y = -3x$

x	y
–3	9
–1	3
1	–3
3	–9



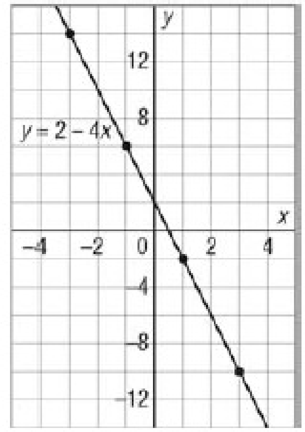
b) $y = 2x$

x	y
–3	–6
–1	–2
1	2
3	6



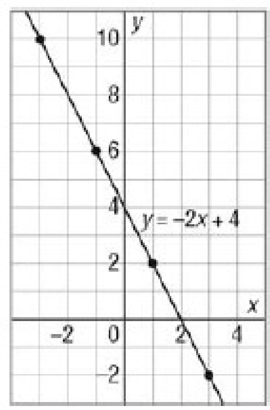
c) $y = 2 - 4x$

x	y
–3	14
–1	6
1	–2
3	–10



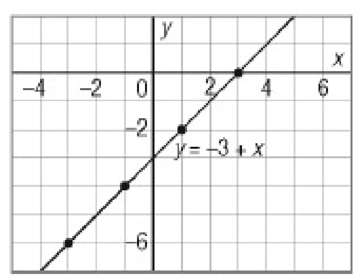
d) $y = -2x + 4$

x	y
–3	10
–1	6
1	2
3	–2



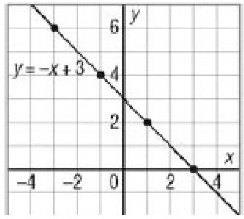
e) $y = -3 + x$

x	y
–3	–6
–1	–4
1	–2
3	0



f) $y = -x + 3$

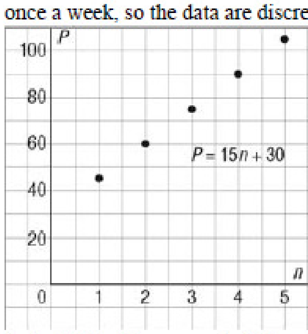
x	y
–3	6
–1	4
1	2
3	0



4. **a)**

Number of Weeks, n	Total Paid, P ($)
1	45
2	60
3	75
4	90
5	105

b) I should not join the points because Alicia pays once a week, so the data are discrete.



c) In the table, P increases by $15 each week. On the graph, to get from one point to the next, move 1 unit right and 15 units up.

5. **a)**

x	y
1	10
2	14
3	18
4	22
5	26

b)

x	y
1	−6
3	−10
5	−14
7	−18
9	−22

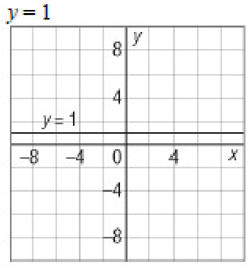
c)

x	y
−2	−15
−1	−9
0	−3
1	3
2	9

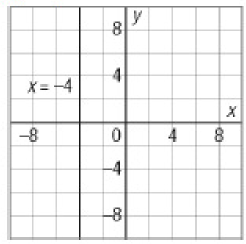
d)

x	y
2	1
4	−2
6	−5
8	−8
10	−11

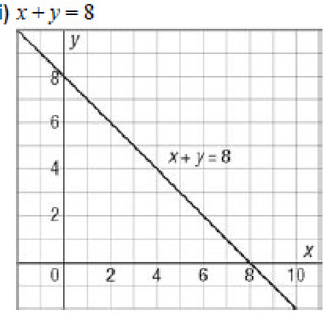
6. **a)** **i)** $y = 1$



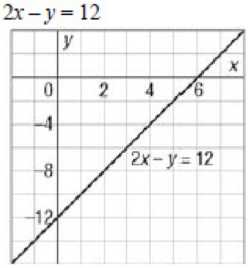
ii) $x = -4$



iii) $x + y = 8$



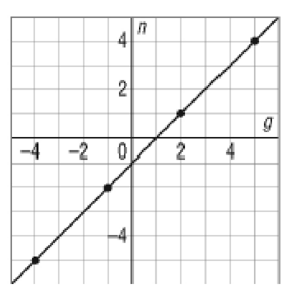
iv) $2x - y = 12$



7. **a)**

g	n
5	4
2	1
−1	−2
−4	−5

b) I would join the points because all values between the plotted points are permitted.



c) $g - n = 1$

4.4 – Matching Equations and Graphs

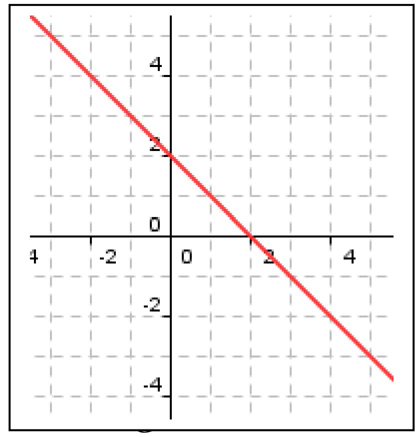
Focus: Match equations and graphs of linear relations.

Warm-up: Use a table of values to match the equations to their corresponding graphs.

1. $y = 2x + 2$

x	*y*
–1	
0	
1	

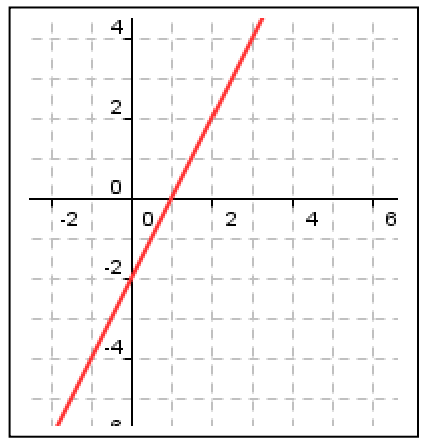
Graph A



2. $x + y = 2$

x	*y*
–1	
0	
1	

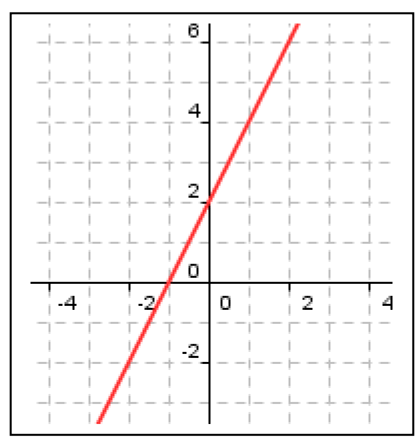
Graph B



3. $y = 2x - 2$

x	*y*
–1	
0	
1	

Graph C

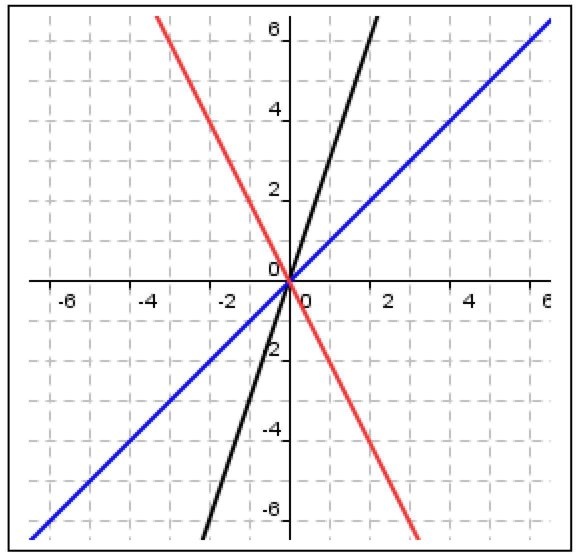


What is the best way to match an equation with its graph?

How do you choose x values to test?

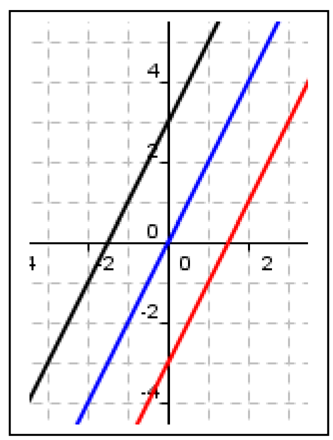
Ex. 1: a) Match each graph with the following equations:

$y = x$ $\qquad$ $y = 3x$ $\qquad$ $y = -2x$



b) Why is testing $x = 0$ unhelpful for this question?

Ex. 2: Which graph on the grid has the equation $y = 2x - 3$?
Justify your choice.



When all lines cross the y axis at a different point, why is it smart to test $x = 0$ into the equation(s)?

HW Assignment
Section 4.4 pg. 188 # 3–9, 11

4.5 – Using Graphs to Estimate Values

Focus: Use interpolation and extrapolation to estimate values on a graph.

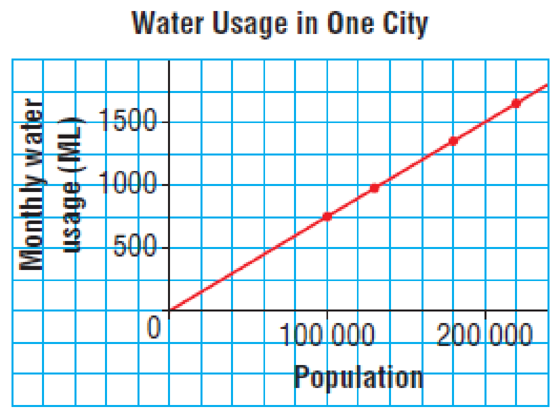
Warm-up: Read over the 'Investigate' and answer the following questions.

Investigate

A city has grown over the past few years. This table and graph show how the volume of water used each month is related to the population.

Population	Monthly Water Usage (ML)
100 000	750
130 000	975
180 000	1350
220 000	1650

1 ML is 1 000 000 L.



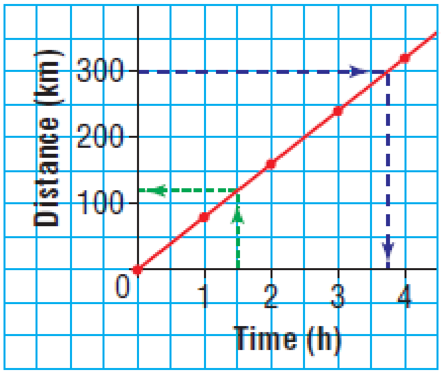
- Estimate the monthly water usage for a population of 150 000 people. ______________
- Estimate the population when the monthly water usage is 1400 ML. ______________
- Predict the water usage for 250 000 people. ______________

What is **interpolation**?

When did you use interpolation in the 'Investigate'?

Look at the graph below to get a visual of interpolation.

Graph of a Car Journey

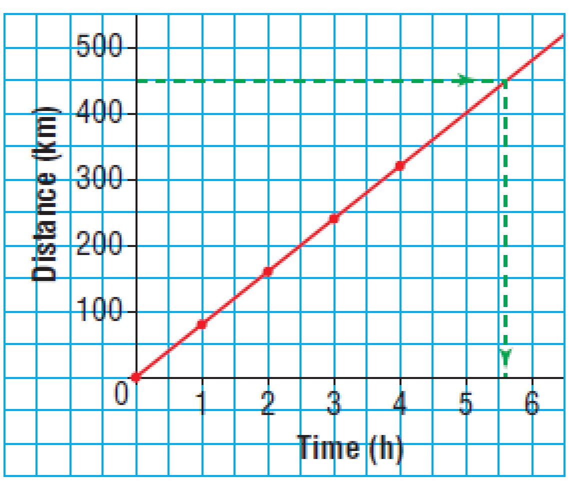


What is **extrapolation**?

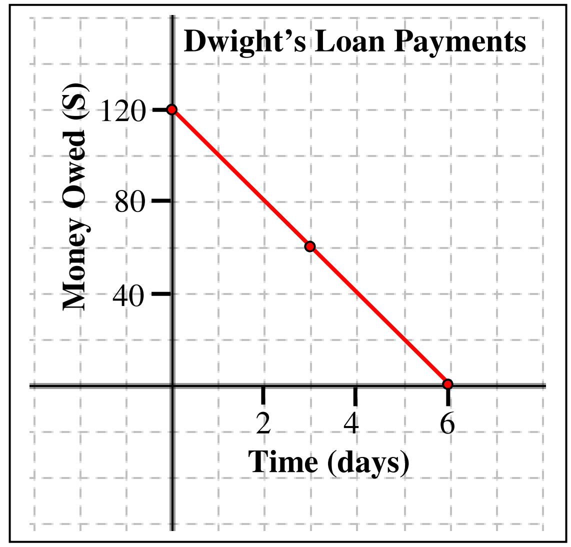
When did you use extrapolation in the Investigate?

Look at the graph below to get a visual of extrapolation.

Graph of a Car Journey



Ex. 1: Dwight borrowed some money from Howard to buy new basketball shoes. He pays some money back each day. The graph shows how the money is repaid over time. The data are discrete because payments are made every day.



a) How much money did Dwight originally borrow?

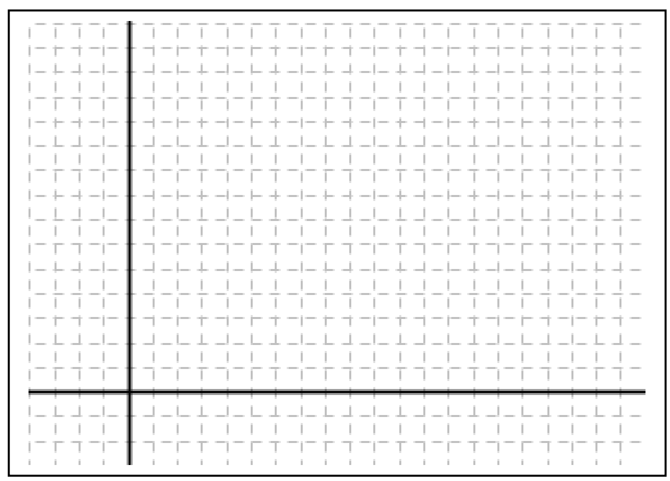
b) How much money does he still owe after 1 day?

c) How many days will it take Dwight to repay half of what he borrowed?

d) Are you interpolating or extrapolating? Explain.

Ex. 2: Corey goes biking.

Every 3 minutes (x), Corey travels 1.5 km (y).

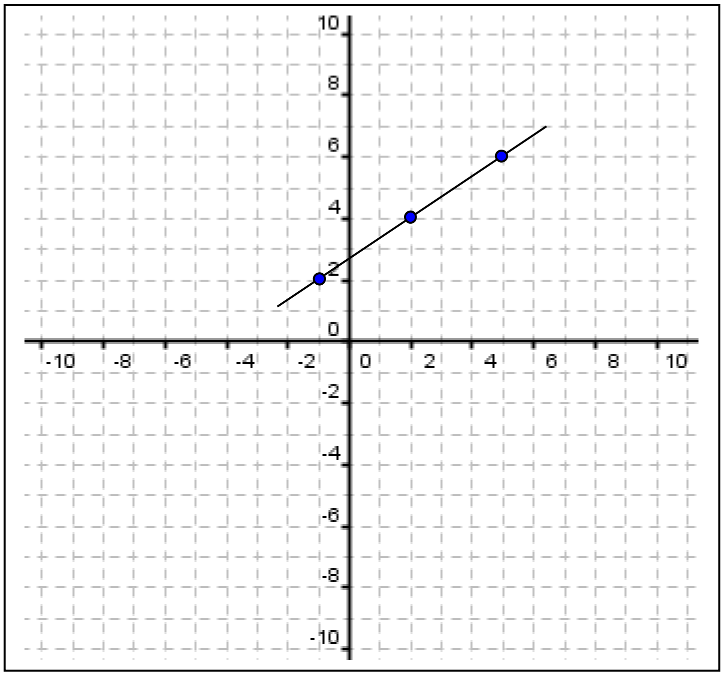


a) Draw a graph for the first 12 minutes of biking, but leave room at the end of the graph.

b) How far has Corey biked after 7 minutes?
Is this interpolation or extrapolation?

c) How far will Corey have biked after 17 minutes? Extend your graph to 18 minutes to assist you.
Is this interpolation or extrapolation?

Ex. 3: Use the graph to answer the following.



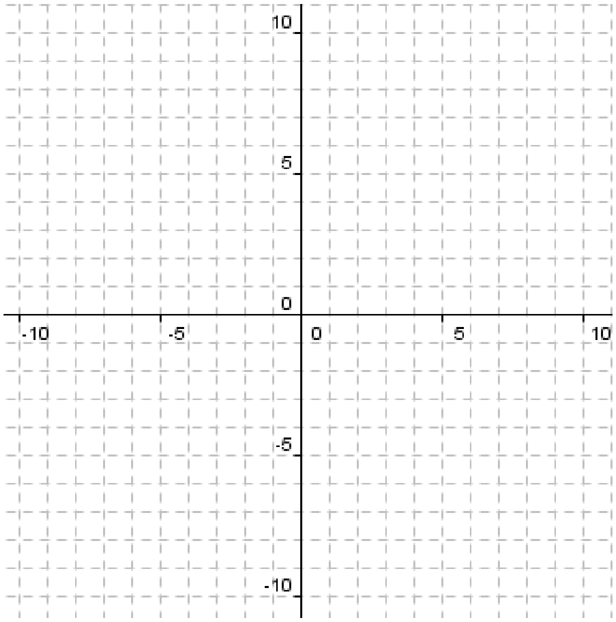
a) Determine the value of x when $y = -2$.
Are you interpolating or extrapolating?

b) Determine the value of y when $x = 4$.
Are you interpolating or extrapolating?

Reflection:
Give one real life example of extrapolation that could have a significant impact on our world.

HW Assignment
Section 4.5 pg. 196 # 4–14

6.



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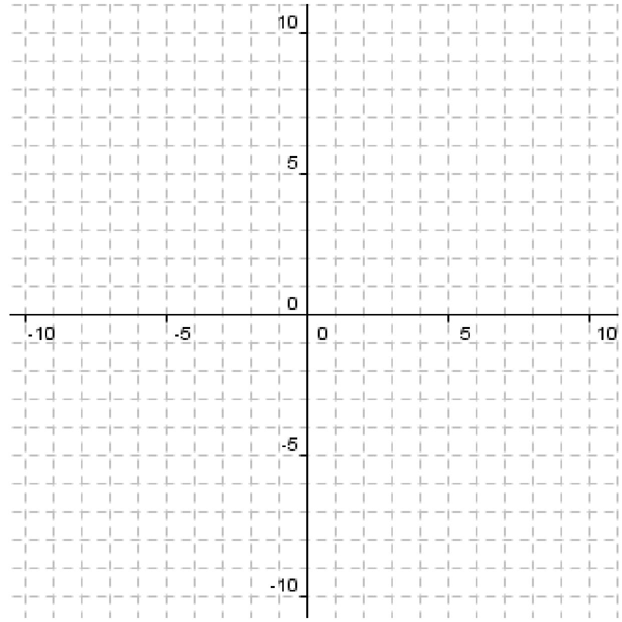
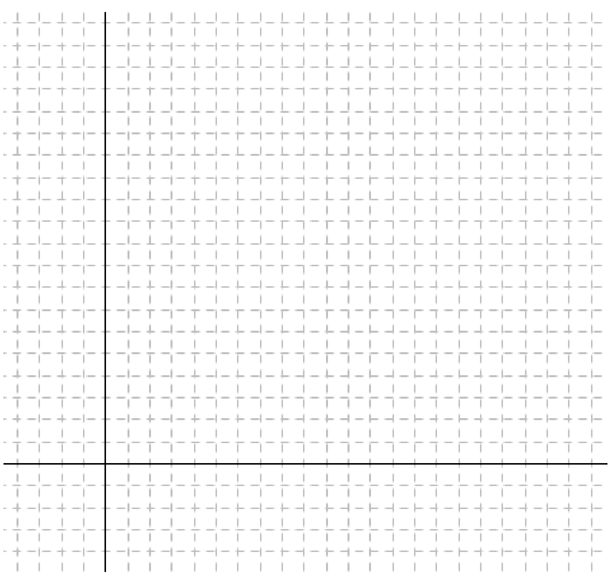
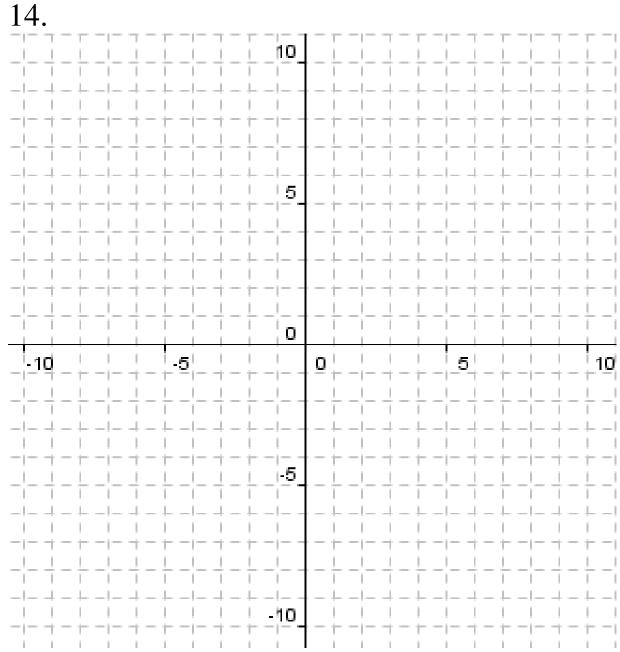


13.



14.



Study Guide

Generalize a Pattern

Term Number, n	Term Value, v	Pattern
1	3	$2(1) + 1$
2	5	$2(2) + 1$
3	7	$2(3) + 1$
:	:	:
n		$2(n) + 1$

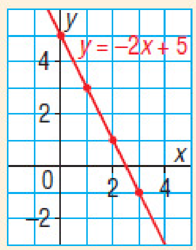
Each term value is 2 more than the preceding term value.
Start with the expression $2n$ and adjust it as necessary to produce the numbers in the table.
The expression is: $2n + 1$
The equation is: $v = 2n + 1$

Linear Relations

- The graph of a linear relation is a straight line.
To graph a linear relation, first create a table of values.
For example, to graph the linear relation: $y = -2x + 5$

x	y
0	5
1	3
2	1

Choose 3 values of x, then use the equation to calculate corresponding values of y.

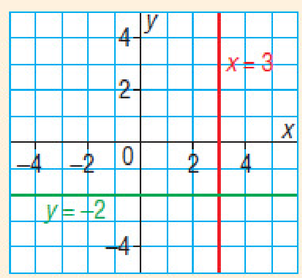


Each point on the graph is 1 unit right and 2 units down from the preceding point.

Another form of the equation of the graph above is $2x + y = 5$.

Horizontal and Vertical Lines

- The graph of the equation $x = a$, where a is a constant, is a vertical line.
The graph of the equation $y = a$, where a is a constant, is a horizontal line.



Interpolation and Extrapolation

- Interpolation is determining data points *between* given points on the graph of a linear relation.

Extrapolation is determining data points *beyond* given points on the graph of a linear relation.
When we extrapolate, we assume that the linear relation continues.

HW Assignment
Chapter Review pg. 201-203 # 1 to 17 All