3.1 – What is a Rational Number?
Math 9

> **Definition of a Rational Number**
>
> A rational number is any number that can be written in the form $\dfrac{m}{n}$,
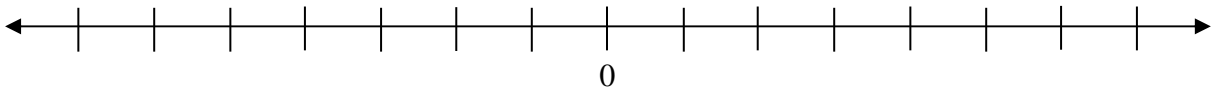>
> Where m and n are integers and $n \neq 0$

Numbers that can be written as fractions are called rational numbers.
The following can be written as fractions, therefore are rational numbers.
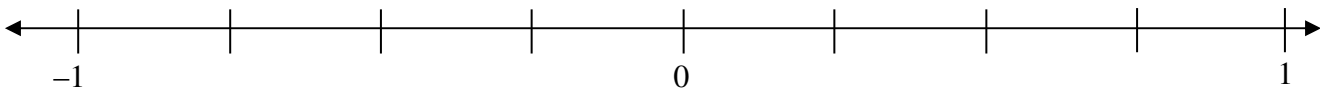
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-

Numbers that can not be written as fractions are called _____.

Label the number line below using integers.

0

> For every positive integer, there is a corresponding negative integer.

Label the number line below using fractions.

−1   0   1

> For every **positive fraction**, there is a corresponding **negative fraction**.
>
> Since any fraction can be written as a decimal,
> for every **positive decimal** there is a corresponding **negative decimal.**

Convert the following decimals to fractions.

a) 0.7   b) 10.2   c) $0.\overline{3}$ d) $8.\overline{5}$

Determine the following quotients.
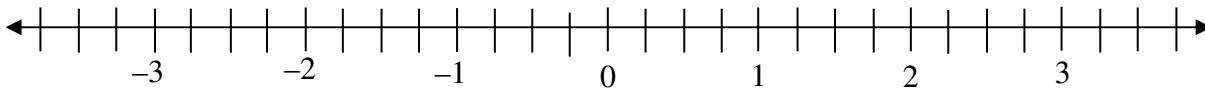
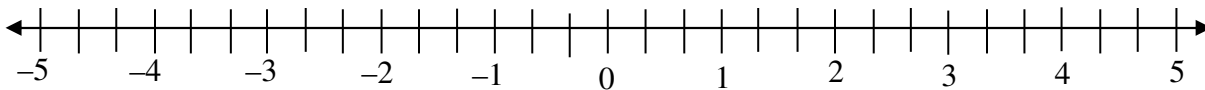a) $\dfrac{-2}{20}$ b) $\dfrac{2}{-20}$ c) $-\dfrac{2}{20}$

What do you notice about the quotients?

Graph the following rational numbers on the number line

a) $3,\ 0.25,\ -1.5,\ \dfrac{3}{2},\ -2\dfrac{3}{4}$

b) $5,\ \dfrac{2}{-3},\ 3\dfrac{1}{3},\ 1.\overline{3},\ -4.\overline{3},\ -2.\overline{6}$

2

Ex. 1: Write 3 rational numbers between each pair of numbers.

a) 2.65 and –3.24
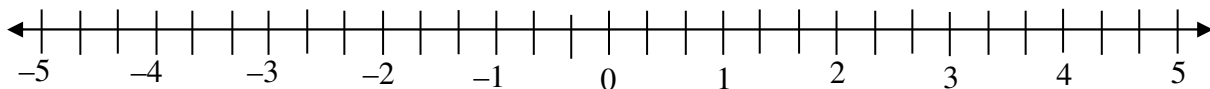
_____

b) –0.21 and –0.22

_____

c) $-\dfrac{1}{2}$ and $\dfrac{1}{4}$

_____

d) $-\dfrac{1}{2}$ and $-\dfrac{1}{4}$
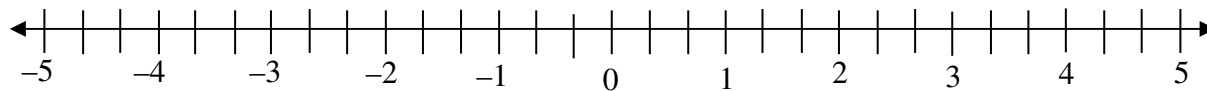
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Ex. 2: Use a number line to order these numbers from least to greatest.

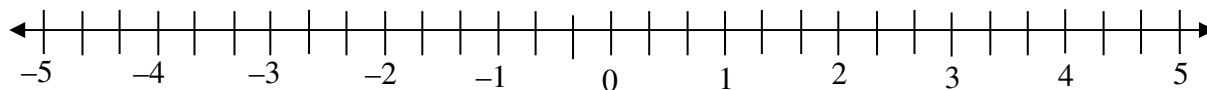$$0.65, \ 2.8, \ -0.7, \ -3.24, \ -0.\overline{7}$$

3

Ex. 3: Order these numbers from greatest to least. Record the numbers on a number line.

$$-\frac{3}{8}, \ \frac{5}{9}, \ -\frac{10}{4}, \ -1\frac{1}{4}, \ \frac{7}{10}, \ \frac{8}{3}$$

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  −5      −4      −3      −2      −1      0       1       2       3       4       5
```

Ex. 4: Order the following rational numbers from least to greatest.
Record the numbers on a number line.

$$1.15, \ -\frac{10}{3}, \ -3.4, \ 2.\overline{7}, \ \frac{2}{9}, \ -2\frac{1}{4}$$

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  −5      −4      −3      −2      −1      0       1       2       3       4       5
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3.2

HW Assignment
Section 3.1   pg. 101   # 5–8, 10, 12acgh, 13, 14aceh, 16–18, 23bc, 24bc, 25
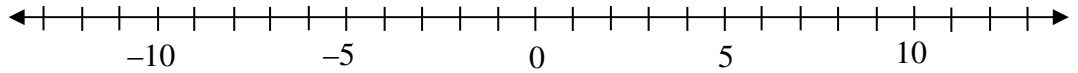
4

Investigate:

Use the number line to illustrate the sum of the following integers.

a) $3 + 7 =$
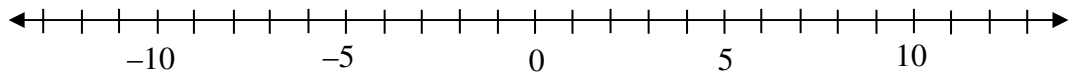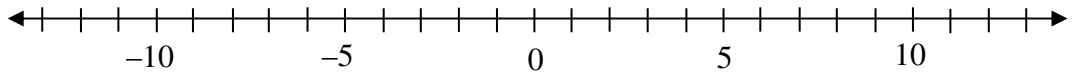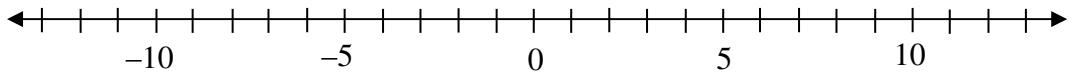
$$\begin{array}{ccccccc} & -10 & & -5 & & 0 & & 5 & & 10 \end{array}$$

b) $-3 + 7 =$

$$\begin{array}{ccccccc} & -10 & & -5 & & 0 & & 5 & & 10 \end{array}$$

c) $3 + (-7) =$

$$\begin{array}{ccccccc} & -10 & & -5 & & 0 & & 5 & & 10 \end{array}$$

d) $-3 + (-7) =$

$$\begin{array}{ccccccc} & -10 & & -5 & & 0 & & 5 & & 10 \end{array}$$
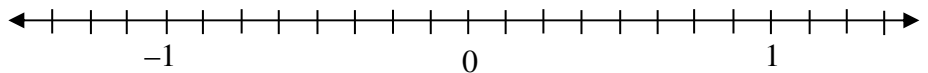
Use the number line to illustrate sum of the following fractions.

a) $\dfrac{3}{8} + \dfrac{7}{8} =$

$$\begin{array}{ccc} -1 & 0 & 1 \end{array}$$

b) $-\dfrac{3}{8} + \dfrac{7}{8} =$

$$\begin{array}{ccc} -1 & 0 & 1 \end{array}$$

c) $\dfrac{3}{8} + \left(-\dfrac{7}{8}\right) =$

$$\begin{array}{ccc} -1 & 0 & 1 \end{array}$$

d) $\left(-\dfrac{3}{8}\right) + \left(-\dfrac{7}{8}\right) =$

$$\begin{array}{ccc} -1 & 0 & 1 \end{array}$$

5

Use the number line to illustrate sum of the following fractions.

a) $1\frac{2}{4} + 2\frac{3}{4} =$

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    -4      -3      -2      -1      0      1      2      3      4
```

b) $-1\frac{2}{4} + 2\frac{3}{4} =$

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    -4      -3      -2      -1      0      1      2      3      4
```

c) $1\frac{2}{4} + \left(-2\frac{3}{4}\right) =$

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    -4      -3      -2      -1      0      1      2      3      4
```
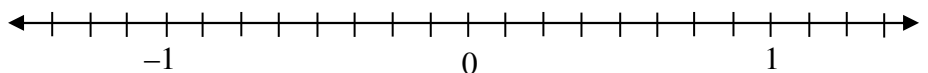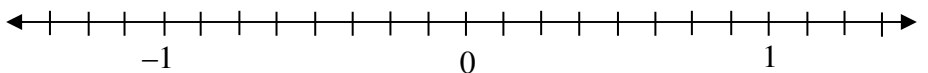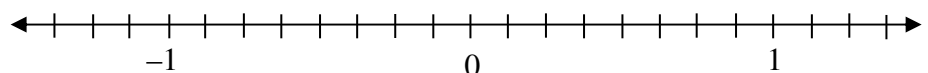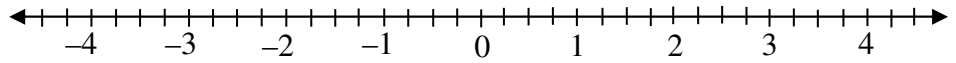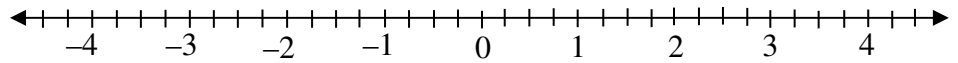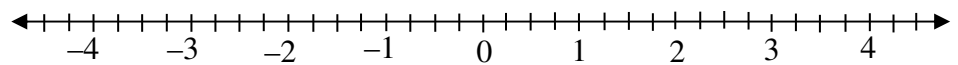
d) $\left(-1\frac{2}{4}\right) + \left(-2\frac{3}{4}\right) =$

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    -4      -3      -2      -1      0      1      2      3      4
```
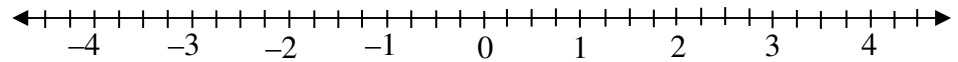
Q. What happens when the fractions have different denominators?

Ex. 1: Find the sum of the following fractions.

a) $\frac{1}{3} + \frac{1}{2}$

b) $\frac{5}{6} + \frac{9}{12}$

c) $2\frac{1}{3} + 1\frac{1}{4}$

6

To **Add** rational numbers in Fraction form:

1) Re-write the mixed numbers as improper fractions.
2) Change the fractions to equivalent fractions.
3) Add the integers in the numerators.
4) Reduce to simplified form.

Ex. 2: Adding Rational Numbers in Fraction and Mixed Number Form

1) $-\dfrac{2}{5} + \dfrac{6}{10} =$

2) $\dfrac{3}{5} + \left(-2\dfrac{1}{3}\right) =$
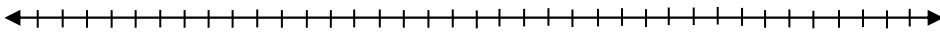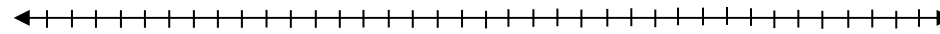
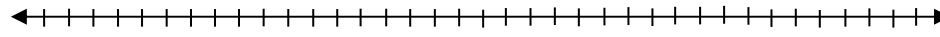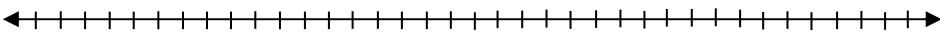3) $-\dfrac{1}{6} + \left(-4\dfrac{2}{3}\right) =$

4) $\left(-3\dfrac{1}{3}\right) + 2\dfrac{5}{6} =$

Ex. 3: Adding Rational Numbers in Decimal Form (Using Number Lines)

1) 3.2 + 1.5 =

2) 2.8 + (– 3.6) =

3) –2.5 + 1.7 =

4) (–2.1) + (–3.5) =

3.3 - Subtracting Rational Numbers
Math 9

Recall:

Subtracting a positive number is equivalent to → _____

Subtracting a negative number is equivalent to → _____

Warm-up: Subtract the following integers.

a) $7 - 3 =$ b) $7 - (-3) =$ c) $-7 - 3 =$ d) $-7 - (-3) =$

We will use the same strategy to subtract rational numbers that we used last day to add rational numbers.

To **Subtract** rational numbers in Fraction form:

1) Re-write the mixed numbers as improper fractions.
2) Change the fractions to equivalent fractions.
3) Subtract the integers in the numerators.
4) Reduce to simplified form.

Subtracting Rational Numbers in Fraction and Mixed Number Form

Ex. 1: Find the difference

a) $\dfrac{4}{5} - \dfrac{1}{4} =$ b) $\dfrac{1}{6} - \dfrac{2}{3} =$ c) $\dfrac{5}{9} - \left(-\dfrac{7}{18}\right) =$ d) $-\dfrac{1}{12} - \dfrac{1}{8} =$

Ex. 2: Predict whether the answer will be positive or negative, then find the difference.
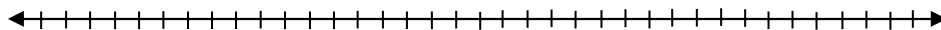
a) $2\dfrac{5}{6} - 3\dfrac{1}{3} =$

b) $-\dfrac{3}{4} - 4\dfrac{1}{6} =$
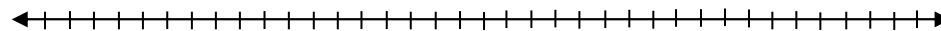
c) $-1\dfrac{1}{4} - \left(-2\dfrac{2}{5}\right) =$

Subtracting Rational Numbers in Decimal Form (Using Number Lines)

Ex. 3: Predict whether the answer will be positive or negative, then find the difference.
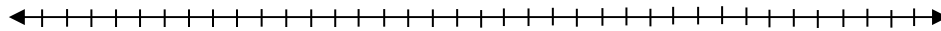
5) $3.2 - 1.5 =$

6) $-1.8 - (-2.2) =$

7) $-2.1 - 1.4 =$

3.1

1. a) Sketch a number line. On the line, place each rational number below.
$-1.3, 2\frac{3}{4}, 1.51, -\frac{8}{5}, -\frac{9}{3}$

b) Which numbers in part a are less than -1.5? Explain how you know.

2. Order the following rational numbers from least to greatest. Place each number on a number line to support your answer.
$-\frac{6}{5}, 1.2, -1.1, -\frac{1}{4}, 0.2, -1\frac{3}{8}$

3. Replace each □ with $<$ or $>$. How could you check your answers?

a) $-\frac{2}{3} \square -\frac{3}{4}$ **b)** $-\frac{8}{3} \square -\frac{9}{4}$

c) $-2.5 \square 0.5$ **d)** $-\frac{4}{5} \square -0.9$

4. Identify a rational number between each pair of numbers. Sketch a number line to illustrate each answer.

a) $1.2, 1.4$ **b)** $-\frac{3}{4}, \frac{5}{8}$

c) $0.4, \frac{1}{3}$ **d)** $-1.05, -\frac{9}{10}$

3.2

5. a) How can you determine the sign of the sum of two numbers before you add them?

b) Determine the sign of each sum, then check by using a calculator.

   i) $2.35 + 3.47$

   ii) $-5.783 + (-0.247)$

   iii) $-\frac{2}{3} + \left(-1\frac{1}{8}\right)$

   iv) $-5.27 + 6.58$

   v) $-\frac{17}{5} + \frac{4}{9}$

   vi) $0.085 + (-0.125)$

6. Determine each sum.

a) $8.37 + 0.58$ **b)** $-21.25 + (-36.57)$

c) $-157.4 + 32.7$ **d)** $\frac{5}{8} + \left(-\frac{1}{9}\right)$

e) $-8\frac{1}{4} + 5\frac{1}{5}$ **f)** $-\frac{5}{3} + \left(-\frac{23}{7}\right)$

3.3

7. The temperature of a freezer changed from $-16.1°C$ to $-14.7°C$.

a) i) By how much did the temperature change?

   ii) Is this an increase or a decrease in temperature? Explain how you know.

b) By how much does the temperature need to change again before it is at $-3.8°C$?

8. Determine each difference.

a) $40.25 - 63.10$ **b)** $-112.2 - (-14.8)$

c) $\frac{2}{5} - \frac{9}{10}$ **d)** $-4\frac{4}{9} - 3\frac{5}{6}$

e) $-1.8 - 4.3$ **f)** $\frac{23}{8} - \left(-\frac{7}{2}\right)$

9. The lowest point on land in North America is Death Valley at 86 m below sea level. The highest point is the peak of Mt. McKinley at 6193.7 m above sea level. How can you use rational numbers to calculate the distance between these two points?
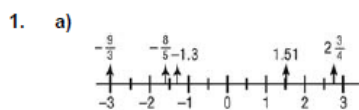
10. a) How can you determine the sign of the difference of two numbers before you subtract them?

b) Determine the sign of each difference, then check by using a calculator.
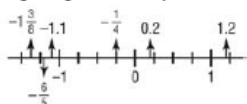
   i) $62.4 - 53.7$ **ii)** $-0.54 - 1.98$

   iii) $\frac{1}{12} - \frac{9}{10}$ **iv)** $5\frac{2}{3} - \left(-7\frac{1}{2}\right)$

Answers:

1. a)

b) $-\frac{9}{3}$, and $-\frac{8}{5}$; they are on the left of -1.5 on the number line.

2. $-1\frac{3}{8}, -\frac{6}{5}, -1.1, -\frac{1}{4}, 0.2, 1.2$

3. a) $>$ **b)** $<$
c) $<$ **d)** $>$

4. Answers will vary. For example:
a) 1.3 **b)** 0
c) $\frac{7}{20}$ **d)** -1

5. a) The sum of two positive numbers is positive. The sum of two negative numbers is negative. The sum of a negative number and a positive number has the same sign as the number farther away from 0.

b) i) Positive; 5.82 **ii)** Negative; -6.03
iii) Negative; $-1\frac{19}{24}$ **iv)** Positive; 1.31
v) Negative; $-2\frac{43}{45}$ **vi)** Negative; -0.04

6. a) 8.95 **b)** -57.82
c) -124.7 **d)** $\frac{37}{72}$
e) $-3\frac{1}{20}$ **f)** $-4\frac{20}{21}$

7. a) i) $1.4°C$ **ii)** An increase
b) $10.9°C$

8. a) -22.85 **b)** -97.4
c) $-\frac{1}{2}$ **d)** $-8\frac{5}{18}$
e) -6.1 **f)** $6\frac{3}{8}$

9. $6193.7 - (-86) = 6279.7$
The distance between the two points is 6279.7 m.

10. b) i) Positive; 8.7 **ii)** Negative; -2.52
iii) Negative; $-\frac{49}{60}$ **iv)** Positive; $13\frac{1}{6}$

3.4 – Multiplying Rational Numbers
Math 9

Warm-up: Multiply the following integers.

a) $4 \times 6 =$

b) $-4 \times 6 =$

c) $4 \times (-6) =$

d) $-4 \times (-6) =$

e) $-9 \times 4 =$

f) $-2 \times (-12) =$

g) $-2 \times 3 \times (-4) =$

Recall:

When two integers have the same sign → _____

When two integers have opposite signs → _____

Warm-up: Multiply the following fractions

a) $\dfrac{1}{2} \times \dfrac{3}{5} =$

b) $\dfrac{2}{3} \times \dfrac{3}{5} =$

c) $\dfrac{4}{9} \times \dfrac{3}{8} =$

d) $\dfrac{1}{8} \times 16 =$

e) $\dfrac{10}{9} \times \dfrac{6}{5} =$

f) $1\dfrac{1}{6} \times \dfrac{3}{21} =$

Recall:

> To **Multiply** rational numbers in Fraction form:
>
> 1) Re-write all numbers as proper or improper fractions.
> 2) Multiply the numerators, multiply the denominators.
> 3) Reduce to simplified form.

We will use the above strategies to multiply rational numbers in fraction form.

Multiplying Rational Numbers in Fraction or Mixed Number Form

Ex. 1: Find the product in reduced form.

a) $-\dfrac{5}{2} \times \dfrac{1}{3} =$

b) $-\dfrac{4}{9} \times \dfrac{3}{-20} =$

c) $\left(-\dfrac{5}{8}\right)\left(-\dfrac{24}{25}\right) =$

d) $\left(\dfrac{2}{3}\right)(-24) =$

e) $\left(-1\dfrac{4}{7}\right)\left(-\dfrac{21}{44}\right) =$

f) $\left(2\dfrac{2}{3}\right)\left(-1\dfrac{3}{4}\right) =$

Multiplying Rational Numbers in Decimal Form

Ex. 2: Find the product.

a) $0.2 \times (-2.5) =$

b) $-1.5 \times 4 =$

c) $(-4.05)(-0.3) =$

HW Assignment
Section 3.4   pg. 127 # 3–7 (odd letters), 9, 11–15

13

Warm-up: Divide the following integers.

a) $12 \div 6 =$ b) $-12 \div 6 =$ c) $12 \div (-6) =$ d) $-12 \div (-6) =$

e) $-24 \div 3 =$ f) $-36 \div (-18) =$ g) $-20 \div 2 \div (-5) =$

Recall:

When two integers have the same sign → _____

When two integers have opposite signs → _____

$6 \div 2 =$ When we divide 6 by 2, this can have two meanings:

     1) _____

     2) _____

$6 \times \dfrac{1}{2} =$ When we multiply 6 by a $\dfrac{1}{2}$, we think to ourselves:

_____

Dividing 6 by 2 and multiplying 6 by $\dfrac{1}{2}$ have the same meaning.

Dividing by the fraction $\left(\dfrac{a}{b}\right)$ is equivalent to → Multiplying by the reciprocal of the fraction $\left(\dfrac{b}{a}\right)$.

To **Divide** rational numbers in Fraction form:

1) Re-write all numbers as proper or improper fractions.
2) Multiply the first fraction by the reciprocal of the second fraction.
3) Reduce to simplified form.

Warm-up: Divide the following fractions.

a) $\dfrac{1}{8} \div 2 =$

b) $\dfrac{4}{9} \div \dfrac{2}{3} =$

c) $2\dfrac{2}{5} \div \dfrac{8}{15} =$

We will use the above strategies to divide rational numbers in fraction form.

Dividing Rational Numbers in Fraction or Mixed Number Form

Ex. 1: Find the quotient in reduced form.

a) $-\dfrac{4}{5} \div \dfrac{1}{4} =$

b) $-\dfrac{4}{9} \div \left(-\dfrac{2}{15}\right) =$

c) $\dfrac{8}{15} \div \left(-\dfrac{24}{25}\right) =$

d) $(-24) \div \dfrac{18}{5} =$

e) $\left(-\dfrac{15}{24}\right) \div \left(-4\dfrac{1}{6}\right) =$

f) $\left(2\dfrac{13}{16}\right) \div \left(-2\dfrac{7}{24}\right) =$

Dividing Rational Numbers in Decimal Form

Ex. 2: Find the quotient.
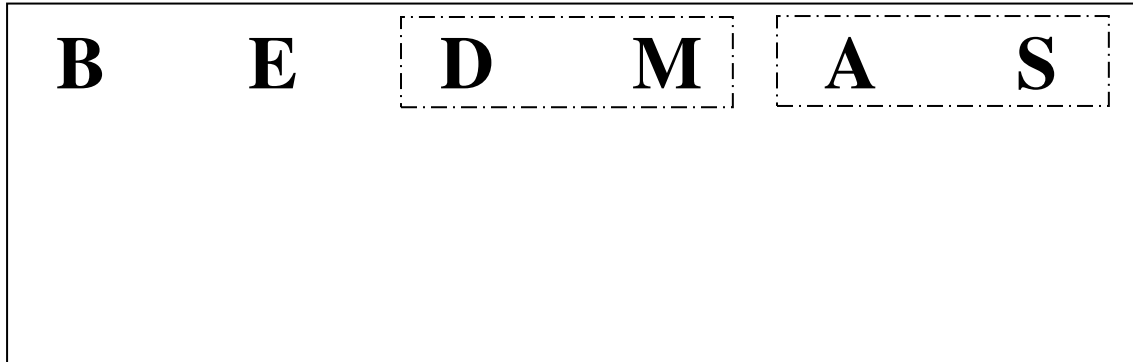
d) $-3.2 \div (-0.4)$

b) $-12.5 \div 0.25$

c) $-0.35 \div 0.7$

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HW Assignment
Section 3.5   pg. 134 # 4, 6, 7acd, 9, 12, 15

</div>

Recall: We learned earlier that integers and fractions are rational numbers.
So the order of operations for all rational numbers is the same as that for integers and fractions.

$$\boxed{\textbf{B} \quad \textbf{E} \quad \boxed{\textbf{D} \quad \textbf{M}} \quad \boxed{\textbf{A} \quad \textbf{S}}}$$

Steps to evaluate
1. Identify the operation to perform according to BEDMAS and underline it.
2. Perform the underlined operation only.
3. Repeat steps 1 & 2 until fully evaluated.

Ex. 1: Evaluate

a) $-0.8 + 1.2 \div (-0.4) \times 2.1$

b) $-4.5 - 2.7 \div \left[-1.1 + 0.8\right]^2$

Ex. 2: To convert Fahrenheit to Celsius, use the formula $C = \dfrac{F - 32}{1.8}$

If the temperature is –4.9° F, what is it in Celsius?

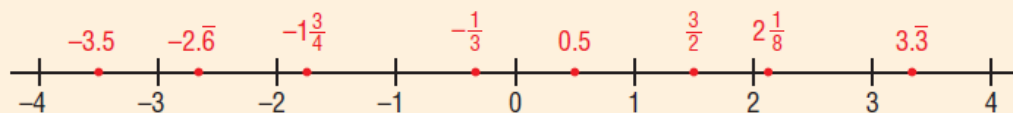**Ex. 3:** Evaluate. Express your final answer as a mixed number.

a) $\dfrac{1}{2} \times \left(-\dfrac{3}{2} \right) - \dfrac{5}{4} \div 1\dfrac{1}{2}$

b) $\left(-\dfrac{1}{2} \right)^2 - \left(-\dfrac{2}{3} \right) \div \left[\dfrac{1}{3} + \left(-\dfrac{3}{12} \right) \right]$

HW Assignment
Section 3.6   pg. 140 # 3–5, 7, 9–13

A rational number is any number that can be written in the form $\frac{m}{n}$, where m and n are integers and $n \neq 0$.

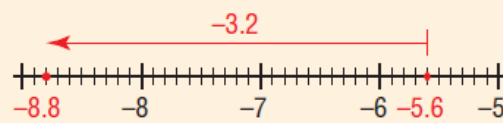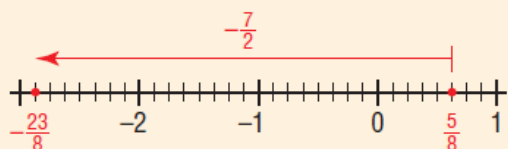This number line illustrates some different forms of rational numbers:

From least to greatest: $-3.5, -2.\overline{6}, -1\frac{3}{4}, -\frac{1}{3}, 0.5, \frac{3}{2}, 2\frac{1}{8}, 3.\overline{3}$

To operate with rational numbers, apply what you know about operating with fractions, decimals, and integers.

- To add rational numbers, visualize a number line.
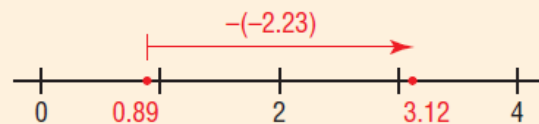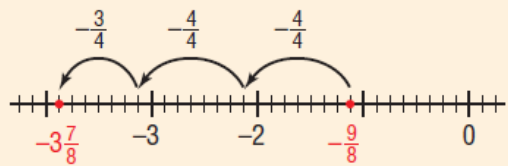
$$\frac{5}{8} + \left(-\frac{7}{2}\right) = -\frac{23}{8} \qquad\qquad (-5.6) + (-3.2) = -8.8$$

- To subtract rational numbers, visualize a number line.

$$-\frac{9}{8} - \frac{11}{4} = -\frac{31}{8} \qquad\qquad 0.89 - (-2.23) = 3.12$$

- To multiply rational numbers, determine the sign of the product first.

$$\left(\frac{3}{4}\right)\left(-\frac{5}{2}\right) = -\frac{15}{8} \qquad \text{and} \qquad (-4.13)(-0.8) = 3.304$$

- To divide rational numbers, determine the sign of the quotient first.

$$\left(-\frac{3}{10}\right) \div \left(-\frac{12}{5}\right) = \frac{1}{8} \qquad \text{and} \qquad 76.63 \div (-7.5) = -10.217\overline{3}$$

The order of operations with rational numbers is the same as the order for whole numbers, fractions, and integers:

- Do the operations in brackets first.
- Then evaluate the exponents.
- Then divide and multiply, in order, from left to right.
- Then add and subtract, in order, from left to right.

HW Assignment
Chapter Review pg. 144-145 # 1 to 23 All