3.1 Factors and Multiples of Whole Numbers

LESSON FOCUS: Determine prime factors, greatest common factors, and least common multiples of whole numbers.

The **prime factorization** of a natural number is the number written as a product of its prime factors.

Example 1: Determining the Prime Factors of a Whole Number
Write the prime factorization of 3300.

CHECK YOUR UNDERSTANDING
Write the prime factorization of 2646. [Answer: $2 \cdot 3^3 \cdot 7^2$]

The **greatest common factor** of two or more numbers is the greatest factor the numbers have in common.

Example 2: Determining the Greatest Common Factor
Determine the greatest common factor of 138 and 198.

CHECK YOUR UNDERSTANDING
Determine the greatest common factor of 126 and 144. [Answer: 18]

The **least common multiple** of two or more numbers is the least number that is divisible by each number.

Example 3: Determining the Least Common Multiple
Determine the least common multiple of 18, 20, and 30.

CHECK YOUR UNDERSTANDING
Determine the least common multiple of 28, 42, and 63. [Answer: 252]

Example 4: Solving Problems that Involve Greatest Common Factor and Least Common Multiple

a) What is the side length of the smallest square that could be tiled with rectangles that measure 16 cm by 40 cm? Assume the rectangles cannot be cut. Sketch the square and rectangles.

b) What is the side length of the largest square that could be used to tile a rectangle that measures 16 cm by 40 cm? Assume that the squares cannot be cut. Sketch the rectangle and squares.

CHECK YOUR UNDERSTANDING

a) What is the side length of the smallest square that could be tiled with rectangles that measure 8 in. by 36 in.? Assume the rectangles cannot be cut. Sketch the square and rectangles. [Answers: a) 72 in.]

b) What is the side length of the largest square that could be used to tile a rectangle that measures 8 in. by 36 in.? Assume that the squares cannot be cut. Sketch the rectangle and squares. [Answers: b) 4 in.]

Homework: Page 140 #3-5, 8-17 – all odd letters

<u>3.2 Perfect Squares, Perfect Cubes, and Their Roots</u>

LESSON FOCUS: Identify perfect squares and perfect cubes, then determine square roots and cube roots.

Make Connections

The edge length of the Rubik's cube is 3 units.

What is the area of one face of the cube? Why is this number a *perfect square*?

What is the volume of the cube? This number is called a **perfect cube**.

Why do you think it has this name?

Competitors solve a Rubik's cube in the world championships in Budapest, Hungary, in October 2007.

Example 1: Determining the Square Root of a Whole Number
Determine the square root of 1296.

CHECK YOUR UNDERSTANDING
Determine the square root of 1764. [Answer: 42]

Example 2: Determining the Cube Root of a Whole Number
Determine the cube root of 1728.

CHECK YOUR UNDERSTANDING
Determine the cube root of 2744. [Answer: 14]

Example 3: Using Roots to Solve a Problem

A cube has volume 4913 cubic inches. What is the surface area of the cube?

CHECK YOUR UNDERSTANDING

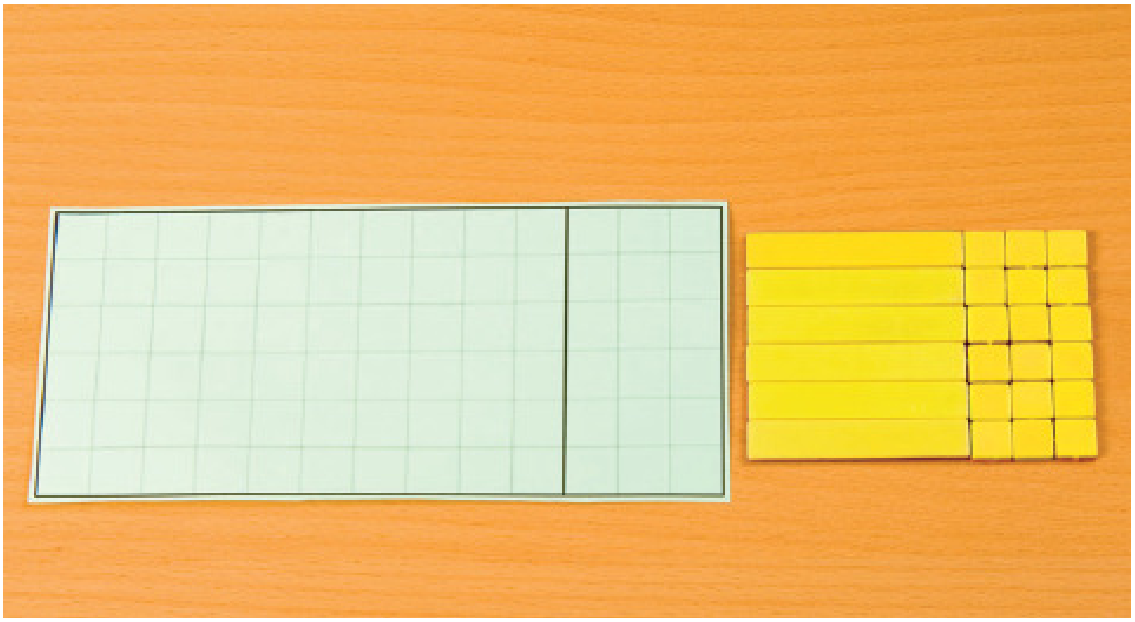
A cube has volume 12 167 cubic feet. What is the surface area of the cube?

[Answer: 3174 square feet]

Homework: Page 146 #4-8, 12-14

3.3 Common Factors of a Polynomial

LESSON FOCUS: Model and record factoring a polynomial.



The variable algebra tile can represent any variable we like. It is usually referred to as the *x*-tile.

Make Connections
Diagrams and models can be used to represent products.

What multiplication sentences are represented above?

What property do the diagrams illustrate?

Example 1: Using Algebra Tiles to Factor Binomials
Factor each binomial.

a) $6n+9$

b) $6c+4c^2$

CHECK YOUR UNDERSTANDING
Factor each binomial. **a)** $3g + 6$ **b)** $8d + 12d^2$ [Answers: a) $3(g + 2)$ b) $4d(2 + 3d)$]

Example 2: Factoring Trinomials
Factor the trinomial: $5-10z-5z^2$.

CHECK YOUR UNDERSTANDING
Factor the trinomial: $6-12z+18z^2$. [Answer: $6(1-2z+3z^2)$]

Example 3: Factoring Polynomials in More than One Variable
Factor the trinomial. Verify that the factors are correct.
$-12x^3y-20xy^2-16x^2y^2$

CHECK YOUR UNDERSTANDING
Factor the trinomial. Verify that the factors are correct.
$-20c^4d-30c^3d^2-25cd$ [Answer: $-5cd\left(4c^3+6c^2d+5\right)$]

Homework: Page 155 #4-10, 12, 14-18 – all odd letters

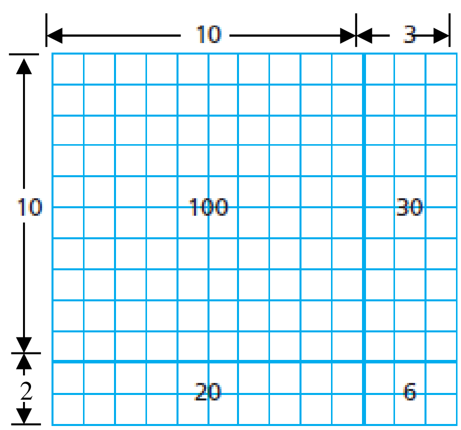
3.4 Modelling Trinomials as Binomial Products

LESSON FOCUS: Explore factoring polynomials with algebra tiles.

Make Connections

We can use an area model and the distributive property to illustrate the product of two 2-digit numbers.

$$\begin{aligned}12 \times 13 &= (10 + 2)(10 + 3)\\ &= 10(10 + 3) + 2(10 + 3)\\ &= 10(10) + 10(3) + 2(10) + 2(3)\\ &= 100 + 30 + 20 + 6\\ &= 156\end{aligned}$$



How could you use an area model to identify the binomial factors of a trinomial?

Construct Understanding

TRY THIS

You will need algebra tiles. Use only positive tiles.

A. Use 1 x^2-tile, and a number of x-tiles and 1-tiles.

- Arrange the tiles to form a rectangle. If you cannot make a rectangle, use additional x- and 1-tiles as necessary. For each rectangle, sketch the tiles and write the multiplication sentence it represents.
- Choose a different number of x-tiles and 1-tiles and repeat until you have 4 different multiplication sentences.

B. Use 2 or more x^2-tiles and a number of x-tiles and 1-tiles.

- Arrange the tiles to form a rectangle. Use additional x- and 1-tiles if necessary. For each rectangle, sketch the tiles and write the corresponding multiplication sentence.
- Repeat with different numbers of tiles until you have 4 different multiplication sentences.

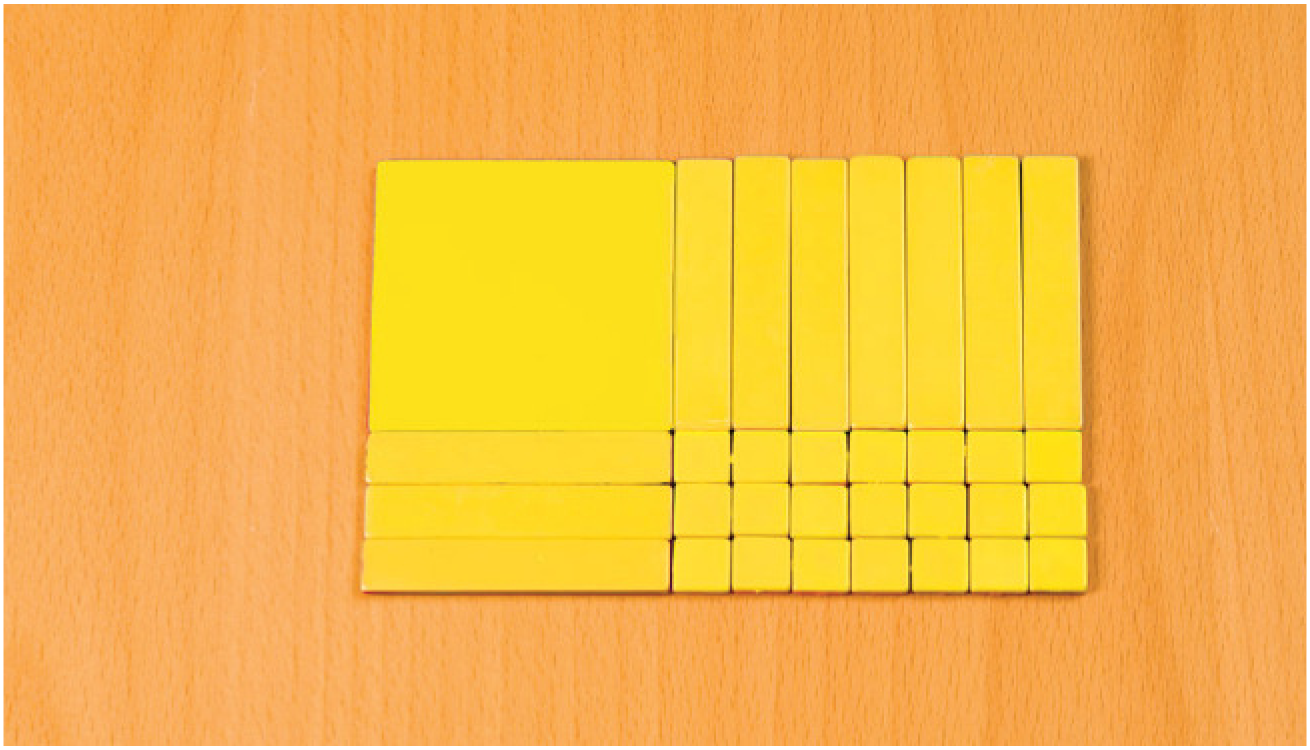
C. Share your work with a classmate. What patterns do you see in the products and factors?

Example 1: Represent x^2+5x+6 by a rectangle using algebra tiles and sketch and label your result.

Homework: Page 158 #1-4

3.5 Polynomials of the Form $x^2 + bx + c$

LESSON FOCUS: Use models and algebraic strategies to multiply binomials and to factor trinomials.



Make Connections

How is each term in the trinomial below represented in the algebra tile model above?

$(c+3)(c+7)=c^2+10c+21$

Example 1: Multiplying Two Binomials
Expand and simplify.
a) $(x-4)(x+2)$
Method 1: rectangular diagram

Method 2: distributive property

b) $(8-b)(3-b)$
distributive property only

CHECK YOUR UNDERSTANDING
Expand and simplify. **a)** $(c+3)(c-7)$ **b)** $(5-s)(9-s)$ [Answers: a) $c^2-4c-21$ **b)** $45-14s+s^2$]

Factoring a Trinomial
To determine the factors of a trinomial of the form $x^2 + bx + c$, first determine two numbers whose sum is b and whose product is c. These numbers are the constant terms in two binomial factors, each of which has x as its first term.

Example 2: Factoring Trinomials
Factor each trinomial.
a) x^2-2x-8

b) $z^2-12z+35$

CHECK YOUR UNDERSTANDING
Factor each trinomial.
a) x^2-8x+7 **b)** $a^2+7a-18$ [Answers: a) $(x-1)(x-7)$ **b)** $(a-2)(a+9)$]

Example 3: Factoring a Trinomial Written in Ascending Order
Factor. $-28-3w+w^2$

CHECK YOUR UNDERSTANDING
Factor. $-30+7m+m^2$ [Answer: $(-3+m)(10+m)$ **or** $(m-3)(m+10)$]

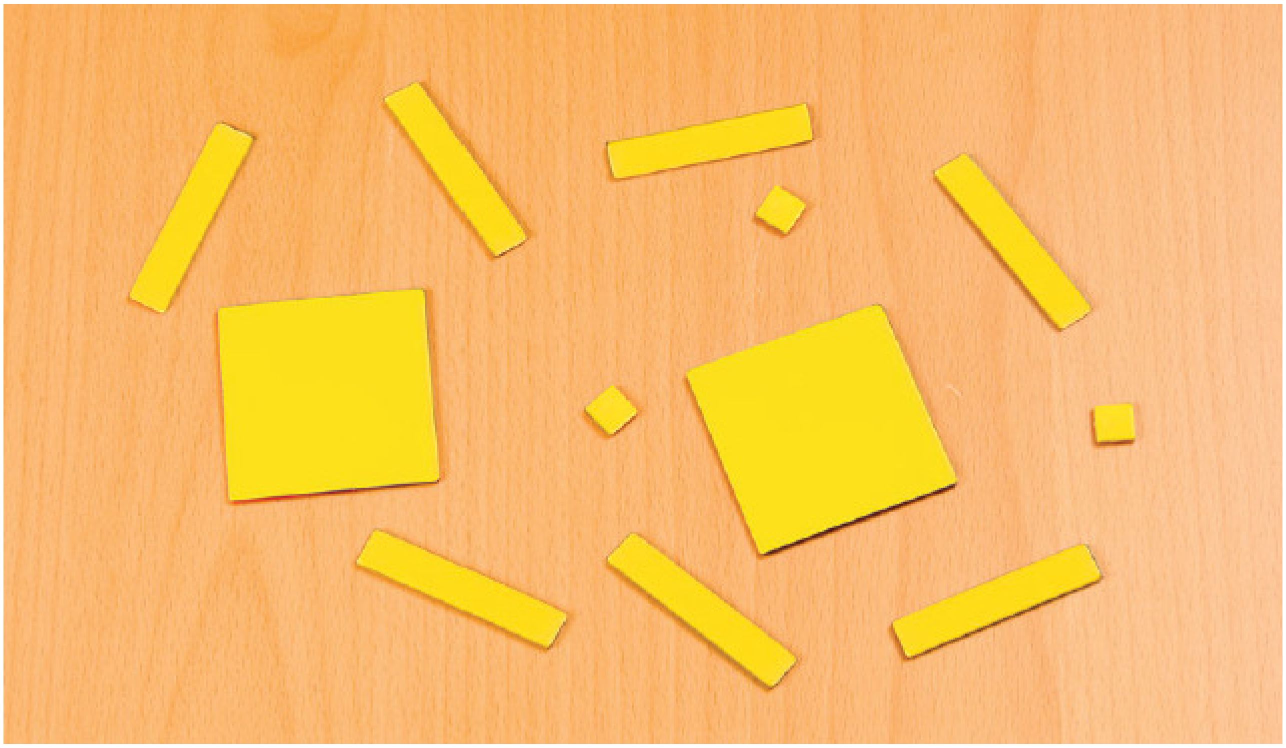
Example 4: Factoring a Trinomial with a Common Factor and Binomial Factors
Factor. $-4t^2-16t+128$

CHECK YOUR UNDERSTANDING
Factor. $-5h^2-20h+60$ [Answer: $-5(h-2)(h+6)$]

Homework: Page 166 #6, 8, 10-15, 21

3.6 Polynomials of the Form $ax^2 + bx + c$

LESSON FOCUS: Extend the strategies for multiplying binomials and factoring trinomials.



Make Connections

Which trinomial is represented by the algebra tiles shown above?
How can the tiles be arranged to form a rectangle?

Example 1: Multiplying Two Binomials with Positive Terms
Expand: $(3d+4)(4d+2)$

Method 1: algebra tiles

Method 2: rectangular diagram

Method 3: distributive property

CHECK YOUR UNDERSTANDING
Expand: $(5e + 3)(2e + 4)$ [Answer: $10e^2 + 26e + 12$]

Example 2: Multiplying Two Binomials with Negative Coefficients
Expand and simplify: $(-2g+8)(7-3g)$

CHECK YOUR UNDERSTANDING
Expand and simplify: $(6t-9)(7-5t)$ [Answer: $-30t^2+87t-63$]

Factoring by decomposition is factoring after writing the middle term of a trinomial as a sum of two terms, then determining a common binomial factor from the two pairs of terms formed.

Consider $(3x+2)(x-10)$**, if we expand and simplify we get**

$$(3x+2)(x-10)=3x(x-10)+2(x-10)$$

$$=3x^2-30x+2x-20$$

-30 and 2 have a sum of –28 and a product of –60, which is the same as the product of 3 and –20.

$$=3x^2-28x-20$$

To factor a trinomial of the form ax^2+bx+c**, look for two integers with a sum of** b **and a product of** ac**.**

Example 3: Factoring a Trinomial by Decomposition

a) $2h^2+7h+6$

sum of b $+$ product of ac $\times$

b) $3p^2-10p+3$

sum of b $+$ product of ac $\times$

c) $3m^2 - 30m + 75$

sum of b product of ac

$+$ $\times$

d) $3t^2 + 14t - 5$

sum of b product of ac

$+$ $\times$

e) $6x^2 - 5x - 6$

sum of b product of ac

$+$ $\times$

f) $15x^2 + 17xy - 4y^2$

sum of b product of ac

$+$ $\times$

CHECK YOUR UNDERSTANDING

Factor. **a)** $8p^2 - 18p - 5$ **b)** $24h^2 - 20h - 24$ [Answers: a) $(2p-5)(4p+1)$ b) $4(2h-3)(3h+2)$]

Example 4: Finding Unknown Coefficients

For which integral value(s) of *h* can $9x^2 + hx + 4$ be factored?

CHECK YOUR UNDERSTANDING

For which integral value(s) of *k* can $25x^2 + kx + 1$ be factored? [Answers: ± 10, ± 26]

Homework: Page 177 #6, 7, 9, 10, 13, 15, 18, 19 – all odd letters

3.7 Multiplying Polynomials

LESSON FOCUS: Extend the strategies for multiplying binomials to multiplying polynomials.

Example 1: Using the Distributive Property to Multiply Two Polynomials
Expand and simplify.

a) $(2h+5)(h^2+3h-4)$

b) $(-3f^2+3f-2)(4f^2-f-6)$

CHECK YOUR UNDERSTANDING
Expand and simplify.

a) $(3k+4)(k^2-2k-7)$

b) $(-2t^2+4t-3)(5t^2-2t+1)$

[Answers: a) $3k^3-2k^2-29k-28$ b) $-10t^4+24t^3-25t^2+10t-3$]

Example 2: Multiplying Polynomials in More than One Variable

a) $(a+b)^2$

b) $(a-b)^2$

In general

$(a+b)^2 =$

Square the first term of the binomial.	**Take the product of the two terms of the binomial and double it.**	Square the last term of the binomial.

c) $(x+7y)^2$

d) $(x-2y)^2$

e) $(x+8)^2$

f) $(4a-5b)^2$

g) $(6p-7)^2$

h) $(3a+1)^2$

i) $\left(3-x\right)^2$

j) $\left(5a+2b\right)^2$

k) $\left(4x-3y\right)^2$

We say that $a^2 + 2ab + b^2$ is a **perfect square trinomial**.

l) $(x+2y)^3$

CHECK YOUR UNDERSTANDING

Expand and simplify.

a) $(4k-3m)^2$ **b)** $\left(2v-5w\right)\left(3v+2w-7\right)$

[Answers: a) $16k^2-24km+9m^2$ b) $6v^2-11vw-14v-10w^2+35w$]

Example 3: Simplifying Sums and Differences of Polynomial Products

Expand and simplify.

a) $(2c-3)(c+5)+3(c-3)(-3c+1)$ **b)** $(3x+y-1)(2x-4)-(3x+2y)^2$

CHECK YOUR UNDERSTANDING

Expand and simplify.

a) $(4m+1)(3m-2)+2(2m-1)(-3m-4)$ **b)** $(6h+k-2)(2h-3)-(4h-3k)^2$

[Answers: a) $-15m+6$ b) $-4h^2-22h+26hk-3k+6-9k^2$]

Homework: Page 186 #6, 7, 8-10 a,c , 11, 15 a,c,e

3.8 Factoring Special Polynomials

LESSON FOCUS: Investigate some special factoring patterns.

Make Connections

The area of a square plot of land is one hectare (1 ha).

1 ha =10 000 m^2

So, one side of the plot has length $\sqrt{10000}$ m = 100 m

Suppose the side length of the plot of land is increased by x metres.

What binomial represents the side length of the plot in metres?

What trinomial represents the area of the plot in square metres?

Recall

In general

$$(a+b)^2 =$$

Square the first term of the binomial.

Take the product of the two terms of the binomial and double it.

Square the last term of the binomial.

We say that $a^2 + 2ab + b^2$ is a **perfect square trinomial**.

Example 1: Factoring a Perfect Square Trinomial

Factor each trinomial. Verify by multiplying the factors.

a) $4x^2+12x+9$ **b)** $4-20x+25x^2$

CHECK YOUR UNDERSTANDING

Factor each trinomial. Verify by multiplying the factors.

a) $36x^2+12x+1$ **b)** $16-56x+49x^2$

[Answers: a) $(6x+1)^2$, b) $(4-7x)^2$]

Example 2: Factoring Trinomials in Two Variables

Factor each trinomial. Verify by multiplying the factors.

a) $2a^2-7ab+3b^2$ **b)** $10c^2-cd-2d^2$

CHECK YOUR UNDERSTANDING

Factor each trinomial. Verify by multiplying the factors.

a) $5c^2-13cd+6d^2$ **b)** $3p^2-5pq-2q^2$

[Answers: a) $(5c-3d)(c-2d)$, b) $(3p+q)(p-2q)$]

<u>Factoring the Difference of Squares</u>

Consider $x^2 - 9$**. This could also be written as** $x^2 + 0x - 9$**.**
Factoring we would look for two integers that add to _____ and multiply to _____.

Factor $x^2 - 9$.

Factoring a Difference of Squares

$$a^2 - b^2 = (\qquad\qquad)(\qquad\qquad)$$

Example 3: Factoring a Difference of Squares
Factor the following:

a) $x^2 - 16$

b) $25t^2 - 81$

c) $49 - 9y^2$

Always remember to factor the common factor first

d) $3x^2 - 27$

e) $8ab^2 - 50ac^2d^2$

f) $(a - 2b)^2 - (2a + b)^2$

If $a^2 - b^2 = (a-b)(a+b)$ then $(a-b)(a+b) = a^2 - b^2$

Example 4: Special Products: Difference of Squares
Expand (might also ask to multiply)

a) $(a+5)(a-5)$

b) $(7x-y)(7x+y)$

c) $(10+4j)(10-4j)$

d) $(k^2+1)(k^2-1)$

e) $(a+5)(a-5)(a^2+25)$

Homework: Page 194 #8, 10-13 odd letters, 15, 18