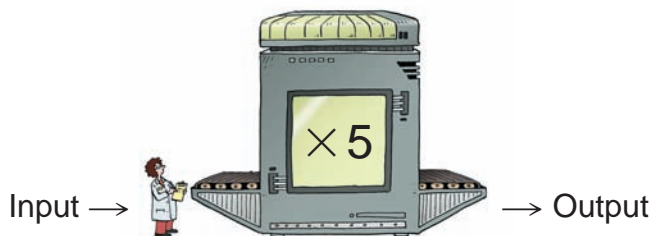# 1

Input/Output Machines

Look at this **Input/Output machine**.
Any number that is put into this
machine is multiplied by 5.
When you input 6, the output is 30.
Suppose you input 9.
What will the output be?

Input → → Output

×5

Explore

➤ Draw your own Input/Output machine.
Choose a number to go inside your machine.
Choose an operation.
Use your machine to create a number pattern.

An *operation* is add,
subtract, multiply, or
divide.

➤ Copy and complete this table of values for your pattern.
Write the pattern rule for the output numbers.

Input	Output
1	
2	
3	

Show *and* Share
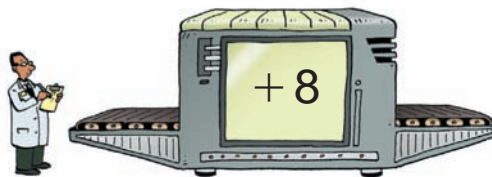
Share your machine and table of values
with another pair of classmates.
Use your classmates' machine
to extend their number pattern.

LESSON FOCUS | Explore the pattern within each column of a table of values.
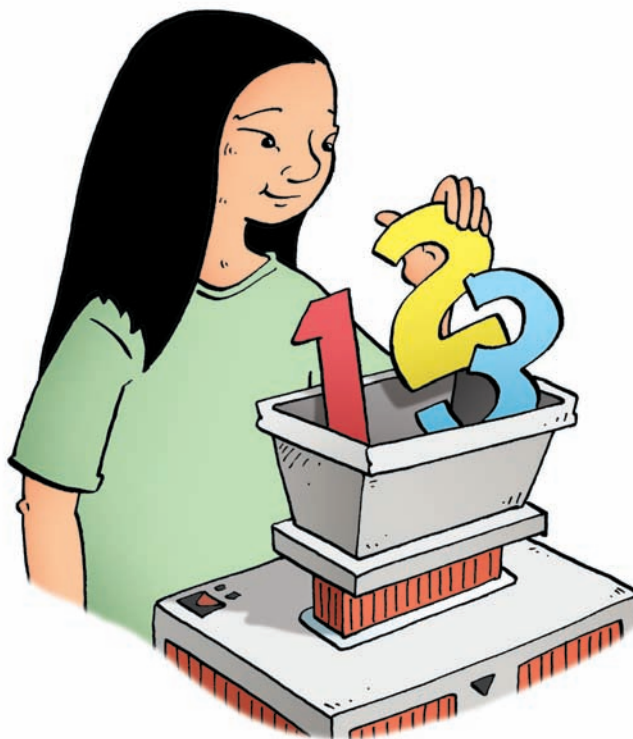
We can use an Input/Output machine to make a growing pattern.

➤ This machine adds 8 to each input to get the output.

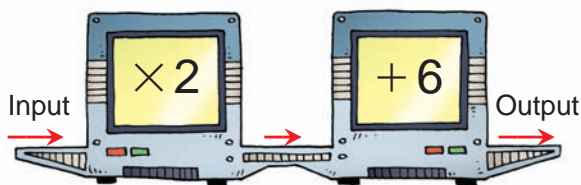The pattern rule that relates the input to the output is: Add 8 to the input.

When each input increases by 1, the output increases by 1.
The pattern rule for the input is: Start at 1. Add 1 each time.
The pattern rule for the output is: Start at 9. Add 1 each time.

Input	Output
1	9
2	10
3	11
4	12

➤ This Input/Output machine doubles each input, then adds 6.

Input ×2 +6 Output

The pattern rule that relates the input to the output is: Multiply the input by 2, then add 6.

Input	Output
2	10
4	14
6	18
8	22

The pattern rule for the input is: Start at 2. Add 2 each time.
The pattern rule for the output is: Start at 10. Add 4 each time.

When each input increases by 2, the output increases by 4.

1. For each Input/Output machine:
   - Copy and complete the table.
   - Write the pattern rule that relates the input to the output.
   - Write the pattern rule for the input.
   - Write the pattern rule for the output.

Input	Output
1	
2	
3	
4	
5	

   a)

   Input ⟶ ×9 ⟶ Output

   b)

   Input ⟶ +12 ⟶ Output

2. For each Input/Output machine:
   - Copy and complete the table.
   - Write the pattern rule that relates the input to the output.
   - Write the pattern rule for the input.
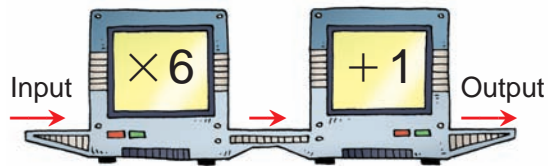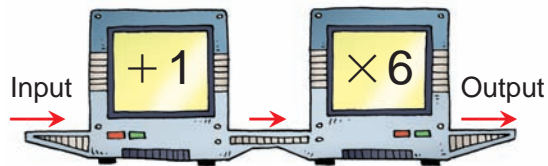   - Write the pattern rule for the output.

Input	Output
2	
4	
6	
8	
10	

   a)

   Input ⟶ ×6 ⟶ +1 ⟶ Output

   b)

   Input ⟶ +1 ⟶ ×6 ⟶ Output

3. Look at question 2 and your tables.
   a) How are the Input/Output machines the same?
      How are they different?
   b) How do the output numbers from the two machines compare? Explain.
   c) Is it possible to get more than one output number for each input? How do you know?

4. Copy and complete this table.

The pattern rule that relates the input to the output is:

Divide the input by 6.

a) Write the pattern rule for the input.

b) Write the pattern rule for the output.

Input	Output
36	
42	
48	
54	
60	

5. Copy and complete this table.

The pattern rule that relates the input to the output is:

Divide the input by 3, then subtract 2.

a) Write the pattern rule for the input.

b) Write the pattern rule for the output.

Input	Output
30	
60	
90	
120	
150	

6. The pattern rule that relates the input to the output is:

Add 4 to the input. Then divide by 2.

Check the data in the Input/Output table.

Identify any output numbers that are incorrect.

How do you know they are incorrect?

Show your work.

Input	Output
4	2
8	4
16	10
26	15
30	19

7. The pattern rule that relates the input to the output is:

Divide the input by 6, then add 5.

a) Check the data in the Input/Output table. Identify any output numbers that are incorrect. How do you know they are incorrect?

b) Correct the table.

c) Write 3 more input and output numbers for this pattern rule.

Show your work.

Input	Output
6	6
12	7
30	10
42	2
54	15

8. The pattern rule that relates the input to the output is:
Multiply the input by 4. Then subtract 3.
Find the missing numbers in the table.
How can you check your answers?

Input	Output
3	9
6	?
9	?
12	45
15	?

9. The pattern rule that relates the input to the output is:
Add 5 to the input. Then multiply by 3.
Find the missing numbers in the table.
What strategies did you use?

Input	Output
2	21
5	?
?	39
11	?
?	57
?	66
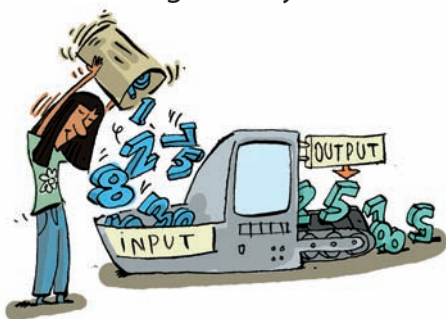
10. a) Draw an Input/Output machine with two operations.
Choose two numbers and two operations for your machine.
b) Choose 5 input numbers.
Find the output numbers.
c) Erase 2 input numbers and 2 output numbers.
Each row must have at least one number.
Trade tables with a classmate.
Trade pattern rules that relate the input to the output.
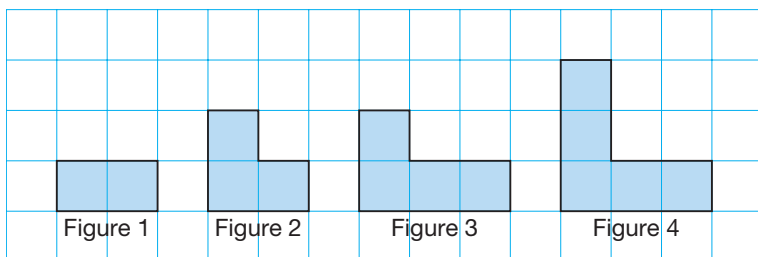Find your classmate's missing numbers.

Reflect

Suppose you want to make an Input/Output machine to convert millimetres to metres.
Describe what your machine would look like.

2

Patterns from Tables

How does this pattern of squares represent the table of values?

Input	Output
1	2
2	3
3	4
4	5

Figure 1   Figure 2   Figure 3   Figure 4

Explore

You will need toothpicks and dot paper.

➤ Build 5 figures to represent the pattern in this table.
Make sure the figures show a pattern.

➤ Draw each figure in the pattern on dot paper.

➤ What patterns do you see in the figures?
In the table?

➤ Write a pattern rule that relates each figure
number to the number of toothpicks.
Predict the number of toothpicks needed
to build the 7th figure.
Use toothpicks to check.

Figure	Number of Toothpicks
1	3
2	5
3	7
4	9
5	11

Show and Share

Compare your patterns and drawings with those of another pair of classmates.
Are your drawings the same or different?
If they are different, do both sets of drawings represent the table of values? Explain.
What Input/Output machine could you use to represent the table?

➤ We can draw pictures to show the relationship
in a table of values.
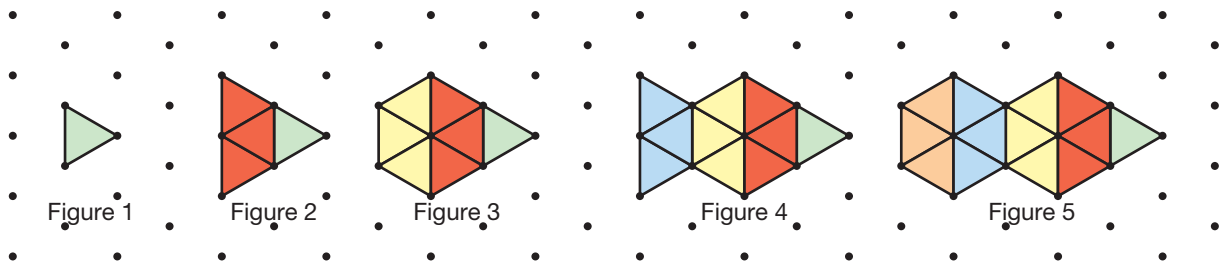
In this table:
The input increases by 1 each time.
The output increases by 3 each time.

Input	Output
1	1
2	4
3	7
4	10
5	13

)3
)3
)3
)3

We could draw a pattern of triangles on
triangular dot paper.
The figure number is the input.
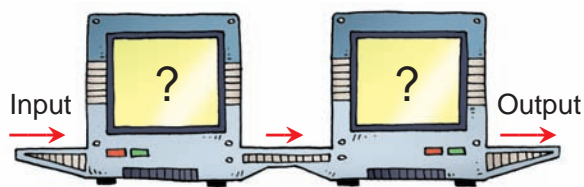The number of triangles in each figure is the output.

Figure 1   Figure 2   Figure 3   Figure 4   Figure 5

➤ We can use a pattern rule to describe the relationship
between the 2 columns in a table of values.
This pattern rule tells us the numbers and operations
in the corresponding Input/Output machine.

The table shows the input and output
for this two-operation machine.

Input   ?   ?   Output

Input	Output
1	1
2	5
3	9
4	13
5	17

)4
)4
)4
)4

To identify the numbers and operations
in the machine:

Think:

The pattern rule for the output is:

When the output increases
by 4, that is a clue
about what to do.

Start at 1. Add 4 each time.

This suggests that the input numbers are multiplied by 4.

Input → ×4   Output →
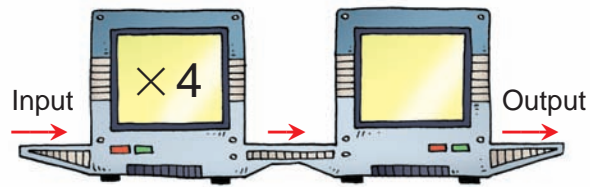
The output increases by 4. Each input must be multiplied by 4.

Look at the input 2.
Multiply by 4.
$2 \times 4 = 8$
But, the output is 5.

Think:

I check all the inputs to make sure I have found the correct numbers and the correct operations.

I have 8. To get 5, I subtract 3.
So, -3 goes into the second part of the machine.
$8 - 3 = 5$

Input → ×4   −3   Output →
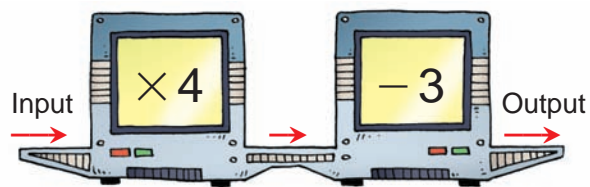
This Input/Output machine multiplies each input by 4, then subtracts 3.
The pattern rule that relates the input to the output is:
Multiply the input by 4.
Then subtract 3.

We can use this rule to predict the output for any input.
For an input of 8, the output should be:
$8 \times 4 - 3 = 29$

We can check this by extending the table.
Add 1 to each input and 4 to each output.

Input	Output
1	1
2	5
3	9
4	13
5	17
6	21
7	25
8	29

4
4
4
4
4
4
4

1. Each table shows the input and output from a machine with one operation. For each table:
   - Identify the number and the operation in the machine.
   - Continue the patterns.
     Write the next 4 input and output numbers.
   - Write the pattern rule that relates the input to the output.

a)

Input	Output
1	7
2	14
3	21
4	28

b)

Input	Output
50	39
49	38
48	37
47	36

c)

Input	Output
2	20
4	40
6	60
8	80

d)

Input	Output
500	485
450	435
400	385
350	335

2. Each table shows the input and output from a machine with two operations. For each table:
   - Identify the numbers and the operations in the machine.
   - Choose 4 different input numbers. Find the output for each input.
   - Predict the output when the input is 10. Check your prediction.
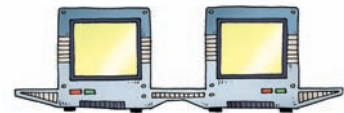
a)

Input	Output
1	2
2	5
3	8
4	11

b)

Input	Output
1	9
2	14
3	19
4	24

c)

Input	Output
3	3
4	5
5	7
6	9

d)

Input	Output
4	17
5	21
6	25
7	29

3. Use the table of values in question 2a.
   Draw pictures to show the relationship in the table.

4. Each table shows the input and output from a machine with two operations.
   - Find the pattern rule that relates the input to the output.
   - Use the pattern rule to find the missing numbers in the table.
   - Use the patterns in the columns to check your answers.
   - Predict the output when the input is 40. Check your prediction.

a)

Input	Output
5	21
6	24
7	27
?	30
9	?
10	?

b)

Input	Output
0	1
5	2
10	3
?	4
20	?
25	?

5. You may need Colour Tiles or counters, and dot paper.
   a) Use tiles, counters, or pictures to show the relationship in this table. Record your work.
   b) Write a pattern rule that relates the input to the output.
   c) Predict the output when the input is 9. Extend your pictures to check.
   d) Which input has an output of 28? Describe the strategy you used to find out.

Input	Output
1	6
2	8
3	10
4	12

6. a) Draw an Input/Output machine with two operations. Choose two numbers and two operations for your machine.
   b) Choose 5 input numbers. Find the output numbers.
   c) Trade tables with a classmate. Find the pattern rule that relates the input to the output. Use this pattern to write the next 4 input and output numbers.

Reflect

When you look at an Input/Output table, what strategies do you use to identify the numbers and operations in the machine?
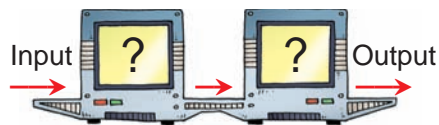
Strategies Toolkit

Explore

Abi made an Input/Output machine that uses two operations.

Here is a table for Abi's machine.

Find out what the machine does to each input number.

Input → ? → ? → Output

Input	Output
15	6
5	4
20	7
25	8
10	5

Show *and* Share
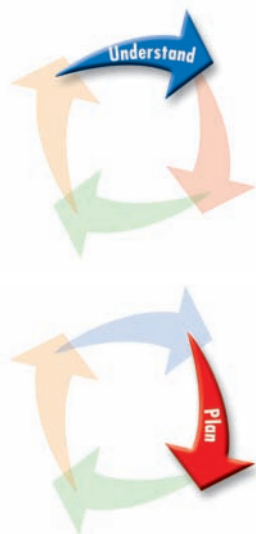
Explain the strategy you used to solve the problem.

Connect

Ben made an Input/Output machine that uses two operations. Here is a table for Ben's machine. What does Ben's machine do to each input number?

Input	Output
2	13
4	23
6	33
8	43
10	53

Strategies

- **Make a table.**
- **Solve a simpler problem.**
- **Guess and test.**
- **Make an organized list.**
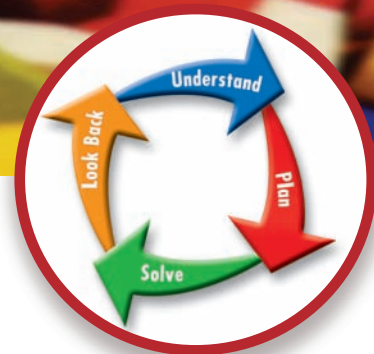- **Use a pattern.**

Understand

What do you know?
- The machine uses two operations on an input number.

Plan

Think of a strategy to help you solve the problem.
- You can **use a pattern**.
- Analyse the pattern in the *Output* column to find out what the machine does to each input number.

The output numbers increase by 10.
This suggests the input numbers are
multiplied by 10. Look at input 2.
Multiply by 10: $2 \times 10 = 20$
But the output is 13.
We subtract 7 from 20 to get 13.

Check: Look at input 4.
Multiply by 10: $4 \times 10 = 40$
Subtract 7: $40 - 7 = 33$
The output should be 23.
This pattern rule does not work.

Try a different pattern.
When the input increases by 2,
the output increases by 10.
So, when the input increases by 1,
the output increases by $10 \div 2 = 5$.
This suggests the pattern involves multiples of 5.
Which two operations does Ben's machine use?

Use the operations in the machine to extend
the pattern of the output numbers.
Check that the rule is correct.

Input	Output
2	13
3	18
4	23
5	28
6	33

Practice

Choose one of the
Strategies

1. Design an Input/Output machine for each table below.
   How did you decide which operations to use?

a)

Input	Output
2	7
4	15
6	23
8	31

b)

Input	Output
3	10
6	19
9	28
12	37

Reflect

Choose one part of question 1.
Explain how you used a pattern to solve it.

What's My Rule?

You will need a set of 10 blank cards for each player.
The object of the game is to be the first player to guess another player's rule.

Before the game begins, each player should:

➤ Label one side of each card "Input" and the other side "Output."
   Label the Input side of each card with the numbers 1 to 10.

➤ Choose a secret rule. You can use one or two operations.
   Write your rule on a separate piece of paper.

➤ Apply your rule to the number on the Input side of each card.
   Write the resulting number on the Output side of that card.

➤ Shuffle your cards. Place them in a pile.

To play:

➤ Player 1 shows all players both sides of her top card.
   Players record the input and output numbers in a table of values.

➤ Player 1 continues to show both sides, one card at a time.
   After each card is shown, Player 1 asks if anyone can guess the rule.
   The player who guesses the rule gets 1 point.
   A player who guessed incorrectly cannot guess again
   until every other player has had a guess.
   If no one guesses the rule after all 10 cards have been shown, Player 1 gets 1 point.

➤ Player 2 has a turn.
   Play continues until all players have shown their cards.