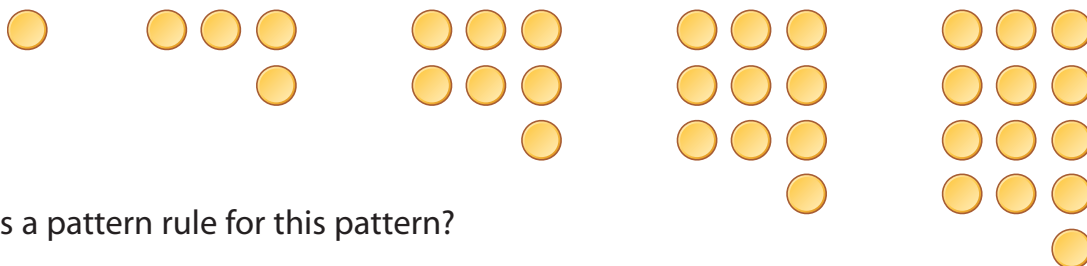# Number Patterns and Pattern Rules

How would you describe this pattern?
What type of pattern is it?

What is a pattern rule for this pattern?

Explore

➤ For each number pattern below:
  Identify a pattern rule.
  Write the next 5 terms.
  What did you do to one term to
  get the next term?
  • 3, 4, 6, 9, 13, . . .
  • 3, 4, 6, 7, 9, . . .
  • 1, 4, 3, 6, 5, 8, . . .
  • 1, 2, 5, 10, 17, 26, . . .

1, 4, 3, 6, 5, 8, ...

➤ Choose one pattern above.
  Use counters to show the pattern
  and to check that the next 2 terms were correct.

➤ Make up a similar pattern.
  Trade patterns with another pair of classmates.
  Write a rule for your classmates' pattern.

Show and Share

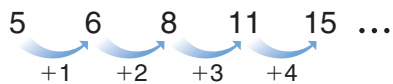Share your patterns with other classmates.
How do you know each pattern rule is correct?
For any pattern, did you find more than one rule? Explain.

LESSON FOCUS **|** Analyse a number pattern and state the pattern rule.

➤ Here is a number pattern.
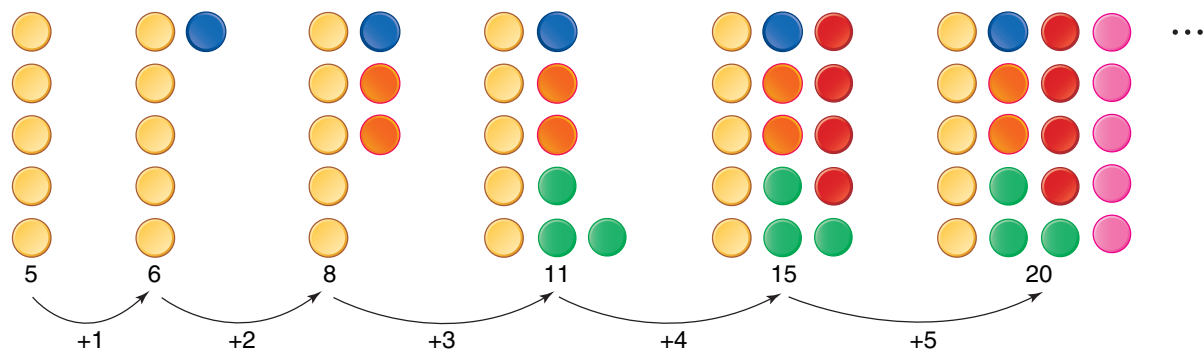
5   6   8   11   15 …
  +1   +2   +3   +4

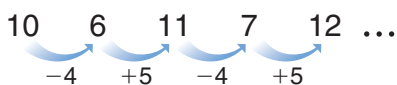Each time, I add 1 more than I added before. This is an increasing pattern.

A pattern rule is:

Start at 5. Add 1.
Increase the number you add by 1 each time.
To get the next 5 terms, continue to
increase the number you add by 1 each time.
5, 6, 8, 11, 15, 20, 26, 33, 41, 50, . . .

We can use counters to show the pattern.

5   6   8   11   15   20   …

  +1   +2   +3   +4   +5

➤ Here is another number pattern.

10   6   11   7   12 …
  −4   +5   −4   +5

I use mental math to subtract and add.

A pattern rule is:

Start at 10. Alternately subtract 4, then add 5.
To get the next 5 terms, continue to subtract 4,
then add 5.
10, 6, 11, 7, 12, 8, 13, 9, 14, 10, . . .

When we alternately subtract, then add, there are two patterns in one.

10, 6, 11, 7, 12, 8, 13, 9, 14, 10, . . .   Add 1

Add 1

1. Write the first 5 terms of each pattern.
   a) Start at 3. Add 2 each time.
   b) Start at 1. Add 2. Increase the number you add by 1 each time.

2. For each pattern in question 1:
   a) Use counters to show the first 5 terms.
   b) Predict the next 2 terms.
   c) Use counters to check your predictions.

3. Write the next 4 terms in each pattern.
   Write each pattern rule.
   What did you do to each term to get the next term?
   a) 1, 2, 4, 5, 7, 8, ...   b) 2, 4, 3, 5, 4, 6, 5, ...
   c) 98, 85, 87, 74, 76, ...   d) 1, 10, 7, 70, 67, 670, ...

4. Find each missing term. Describe the pattern.
   a) 3, 23, 13, 33, □, 43, 33, ...
   b) 99, 98, 198, 197, □, 296, 396, ...
   c) 2, 22, 12, 132, 122, 1342, □, ...

5. What is the 7th term of this pattern?
   Start at 200. Subtract 8 each time.
   How could you find the 7th term without
   writing the first 6 terms?

6. What is the 10th term of this pattern?
   Start at 13. Alternately subtract 4, then add 5.

7. The first 2 terms of a pattern are 6, 12, ...
   How many different patterns can you write with these 2 terms?
   For each pattern, list the first 6 terms and write the pattern rule.
   Show your work.

Reflect

How do you find the pattern rule for a number pattern?
Use an example to explain.

Using Patterns to Solve Problems

What are the missing numbers?
How do you know?

Explore

Sam charges $6 for each hour he baby-sits.

➤ How much does Sam earn when he works
2 hours? 3 hours? 4 hours? 5 hours?
Show your results in a table.

Time Worked (hours)	Money Earned ($)
1	6

➤ What patterns do you see in the table?
How is each term different from the term before?
Use the patterns to predict how
much Sam will earn working 21 hours.

➤ Will Sam earn exactly $40? $45? $50?
How do you know?

➤ Sam saves all the money he earns.
He needs $250 to buy a mountain bike.
How many hours does Sam need
to work?

➤ Make up your own problem you can
solve using this table.
Trade problems with another pair of classmates.
Solve your classmates' problem.

Show and Share

Share your answers with your classmates.
Did you solve the problems the same way? Explain.

One puzzle book costs $17.

➤ How much does it cost to buy 2 books? 3 books? 4 books?

Make a table.
When you add 1 to the
number of books,
you add $17 to the cost.

Number of Books	Cost ($)
1	17
2	34
3	51
4	68

These numbers are multiples of 17.

Two books cost $34.
Three books cost $51.
Four books cost $68.

➤ Use a pattern to predict the cost of 20 books.

One pattern rule for the cost is:

Start at 17. Add 17 each time.

Another pattern rule for the cost is:

The number of books multiplied by 17

To predict the cost of 20 books, multiply: $20 \times 17 = 340$
Twenty books cost $340.

➤ Suppose you have $200.
Can you buy puzzle books and have no money left over?

Extend the pattern to see if 200 is a term.
Use a calculator.

Continue to add 17:
17, 34, 51, 68, 85, 102, 119, 136, 153, 170, 187, 204, …

Two consecutive terms are 187 and 204.

When one number follows another number, the numbers are **consecutive**.

So, 200 is not a term in the pattern.
If you try to spend $200, you will
have money left over.

1. Here is a pattern of linking cubes.
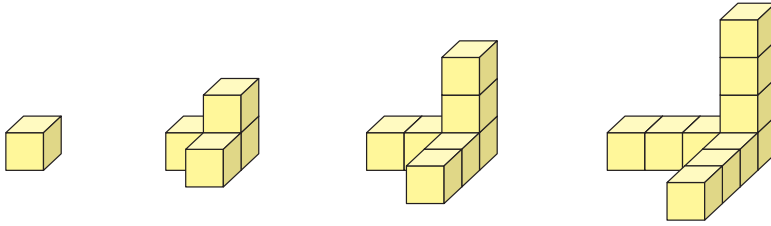
Object 1   Object 2   Object 3   Object 4

The pattern continues. Use linking cubes.
a) Make the next two objects.
b) Copy and complete this table for the
   first 6 objects.

Object	Number of Cubes
1	1

c) How does the pattern grow?
   Write a pattern rule for the number of cubes.
d) How many cubes will there be in the 10th object?
   How do you know?
e) Will any object have 50 cubes? 51 cubes? How do you know?

2. The pattern in this table continues.

Number of CDs	Cost ($)
1	16
2	32
3	48
4	64
5	

Math Link

Nature

In a beehive, bees
make honeycomb to
store their honey.
The cells in the
honeycomb form
a pattern.

a) Which of these numbers is the
   next term in the *Cost* column?
   77, 78, 79, 80, 81
   How do you know?
b) Write a pattern rule for the cost in dollars.
c) Write the next 5 terms in the *Cost* column.
d) How is each term in the *Cost* column different from the term before?
   How is each term different from the following term?

3. Hilary delivers newspapers. Each week she collects $25.
   a) How much money has Hilary collected at the end of 1 week? 2 weeks?
   b) Make a table to show the amounts for the first 8 weeks.
   c) How is each amount different from the amount before?
   d) How much will Hilary collect in total in 3 weeks?
   e) Will Hilary ever collect a total of $240? $250? $260?
      How do you know?
   f) Write a problem you could solve using the table in part b.
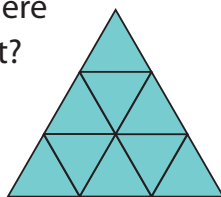   Solve your problem.

4. The sunflower is the only single flower
   that grows as high as 300 cm.
   Suppose it grows 30 cm each week.
   In which week could a sunflower reach a height of 300 cm? Explain.

5. Dave read 40 pages on Monday, 37 pages on Tuesday,
   and 34 pages on Wednesday.
   This pattern of pages read continued until Dave finished his book.
   a) Which of the numbers below is the number of pages
      Dave read on Thursday? How do you know?
      29, 30, 31, 32, 33
   b) What was the total number of pages Dave read the first 7 days?
   c) Dave finished his book on the day he read 1 page.
      How many pages are in the book?
   Show your work.

6. Look at this shape.
   a) How many triangles are there
      with a side length of 1 unit?
      2 units? 3 units?
   b) How many triangles
      are in this shape?

Reflect

How can using patterns
help you solve problems?
Use an example from this
lesson to explain.

At Home

What number patterns
do you see at home?
Look through magazines,
newspapers, and around your
community.
Write about the patterns you see.
How is each term different from the
term before?

Using a Variable to Describe a Pattern

Explore

You will need green Pattern Blocks
and triangular dot paper.
The side length of the block is shown.

➤ Make an increasing pattern with the blocks.
   Draw each figure in the pattern on dot paper.

➤ What is the perimeter of each figure?

➤ Copy and complete this table for the first 3 figures.

| Figure Number	Perimeter (units)
1	

➤ Continue the pattern.
   Make the next 3 figures.
   Draw these figures on dot paper.
   Extend the table for these 3 figures.

➤ What patterns do you see in the table?
   How is each perimeter different from
   the perimeter before?
   How is the perimeter related to the
   figure number?

Show and Share

Compare your table with that of another pair of students.
Suppose you know the figure number.
What would you do to get the perimeter of the figure?
What is the perimeter of the 100th figure? The 200th figure?

➤ Here is a pattern of line segments drawn on dot paper.

Figure 1

Figure 2

Figure 3
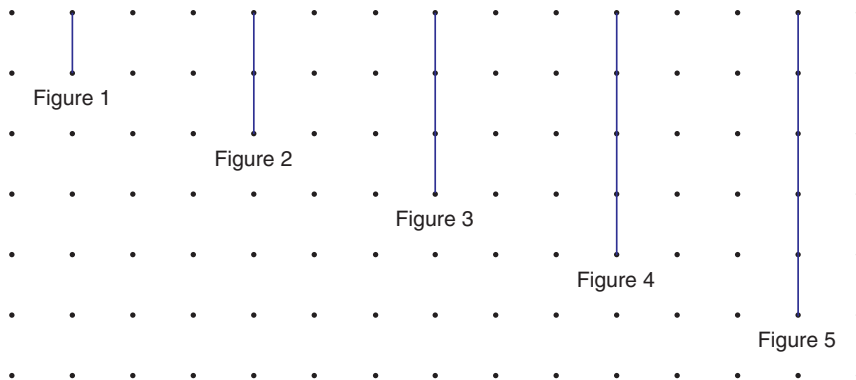
Figure 4

Figure 5

The table shows each figure number and the number of dots on the figure.

Figure Number	Number of Dots
1	2 = 1 + 1
2	3 = 2 + 1
3	4 = 3 + 1
4	5 = 4 + 1
5	6 = 5 + 1

The number of dots is 1 more than the figure number.

We can write each number of dots as this sum: Figure number + 1

We can use a letter, such as f, to represent any figure number.
f is called a **variable**.
Then, the number of dots on Figure f is $f + 1$.
$f + 1$ is an **expression** that represents the pattern in the numbers of dots.
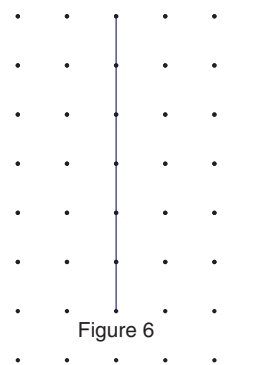
A variable is shown in italics.

We can check that this expression is correct.
For the number of dots on the 6th figure, replace f with 6.
Then, $f + 1 = 6 + 1$
$\qquad\qquad = 7$
The 6th figure has 7 dots.
We continue the pattern above to verify this.

Figure 6

➤ We can use a variable to write a pattern rule.

Look at this pattern: 7, 8, 9, 10, 11, . . .

Each term is 1 more than the preceding term.

Look for a way to relate the value of a term to its position in the pattern.

Term Position	Term Value
1	7 = 1 + 6
2	8 = 2 + 6
3	9 = 3 + 6
4	10 = 4 + 6
5	11 = 5 + 6

Let n represent any term position. Then, the term value is $n + 6$. So, an expression for the pattern rule is $n + 6$.

We can check that the expression $n + 6$ is correct.

For the 5th term, replace n with 5.

$n + 6 = 5 + 6$

$\qquad = 11$

This matches the value of the 5th term in the table above.

So, the expression is correct.

Practice

1. For the pattern below:
   a) Copy and complete the table.
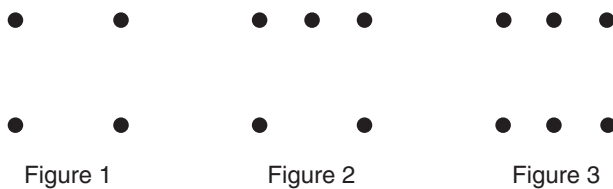   b) Write an expression to represent the pattern in the numbers of dots.

Figure Number	Number of Dots

Figure 1   Figure 2   Figure 3   Figure 4   Figure 5

2. For the pattern below:
   a) Copy and complete the table.
   b) Write an expression to represent the pattern in the numbers of squares.

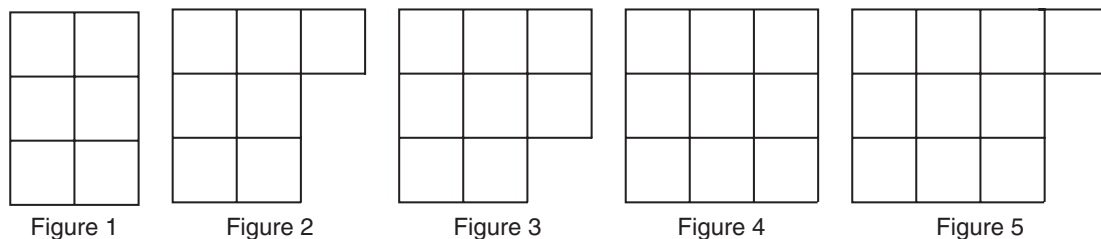| Figure Number	Number of Squares

Figure 1   Figure 2   Figure 3   Figure 4   Figure 5

3. For each table, write an expression for the number of dots in any figure.
Check that each expression is correct.

a)

Figure Number	Number of Dots
1	5
2	6
3	7
4	8
5	9
6	10

b)

Figure Number	Number of Dots
1	6
2	7
3	8
4	9
5	10
6	11

c)

Figure Number	Number of Dots
1	0
2	1
3	2
4	3
5	4
6	5

4. Use a variable to write a pattern rule for each number pattern.
   a) 2, 3, 4, 5, 6, 7, . . .   **b)** 10, 11, 12, 13, 14, 15, . . .   **c)** 8, 9, 10, 11, 12, 13, . . .

5. Find the 100th term in each pattern in question 4.
Explain how you did this.

6. Write an expression for each number pattern.
Write the next 5 terms in each pattern.
Explain how you know the expressions and terms are correct.
   a) 15, 16, 17, 18, 19, . . .   **b)** 16, 17, 18, 19, 20, . . .
Show your work.

7. Here are some decreasing patterns.
Match each pattern with an expression below.
How can you check that you are correct?
   a) 99, 98, 97, 96, 95, . . .   **b)** 34, 33, 32, 31, 30, . . .   **c)** 50, 49, 48, 47, 46, . . .
   A. $51 - t$ **B.** $35 - t$ **C.** $100 - t$

8. Use a variable to write a pattern rule for each number pattern.
   a) 10, 9, 8, 7, 6, 5, . . .   **b)** 40, 39, 38, 37, 36, 35, . . .   **c)** 1000, 999, 998, 997, 996, . . .
How is each pattern different from the patterns in question 4?

Reflect

How can using a variable help you represent a pattern?
Use words, numbers, or pictures to explain.

Tic-Tac-Toe Challenge

You will need 1-cm grid paper.
Think about the game Tic-Tac-Toe.
On a 3 by 3 grid, people take turns
to write X or O.
The winner is the person who gets
3 in a row, column, or diagonal.

Try Tic-Tac-Toe on a 4 by 4 grid.
Take turns to write X or O in a grid
square until one person gets 3 in a row.

Play the game several times.
Try to find a strategy so the person
who plays first always wins.
Where does that person write her first X or O?

Variation: Play Tic-Tac-Toe on a 4 by 4 grid so the first person to
get 4 in a row *loses*.

Strategies Toolkit

Explore

Two students stretch a piece of modelling clay until it breaks into 2 pieces. This is Round 1.
The students then stretch each new piece until it breaks into 2 pieces. This is Round 2.
This process continues.
How many pieces of clay will there be after Round 8?

Show and Share

Describe the strategy you used to solve the problem.

Connect

Suppose a cow produces her first female calf when she is 2 years of age.
After that, she produces a female calf each year.
Suppose each cow produces her first female calf when she is 2 years of age and no cows die.
How many cows will there be after 5 years?

Understand

What do you know?
- Each cow produces a female calf at age 2.
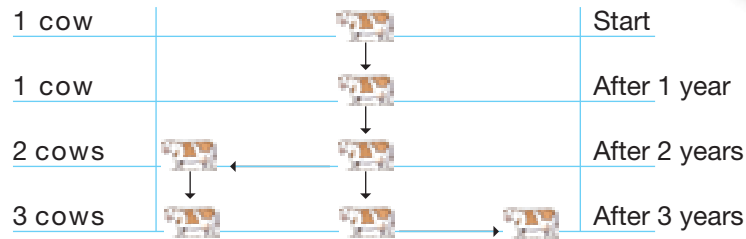- Every year after that, she produces 1 female calf.
- No cows die.

Plan

Think of a strategy to help you solve the problem.
- You can **draw a diagram**.
- Find out how many cows there are after 1 year, then after 2 years, and so on.

Strategies

- **Make a table.**
- **Use a model.**
- **Draw a diagram.**
- **Solve a simpler problem.**
- **Work backward.**
- **Guess and test.**
- **Make an organized list.**
- **Use a pattern.**

LESSON FOCUS | Interpret a problem and select an appropriate strategy.

Copy and continue the diagram.

1 cow		🐄 ↓	Start
1 cow		🐄 ↓	After 1 year
2 cows	🐄 ↓ ◄	🐄 ↓	After 2 years
3 cows	🐄	🐄 ►	🐄 After 3 years

After 1 year, there is 1 cow.
After 2 years, there are 2 cows.
After 3 years, there are 3 cows.
How many cows are there
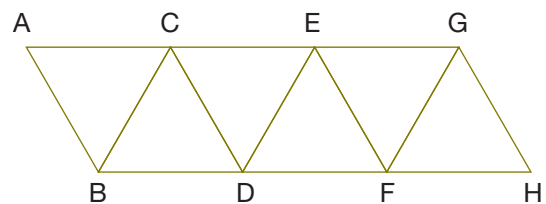after 5 years?

Check your work.
What pattern do you see in the numbers of cows?

Practice

1. A mouse crawls through this maze.
   The mouse always moves forward.
   a) How many different paths could
      the mouse take from A to B?
      From A to C? From A to D?
      What pattern do you see?
   b) Predict the number of different paths the mouse could take from A to H.

2. Here is a regular pentagon. Copy the pentagon.
   Join each vertex to all other vertices.
   How many different triangles are there?
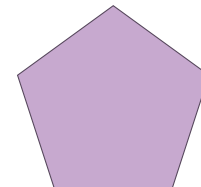
Reflect

How does drawing a diagram help to solve a problem?
Use words, pictures, and numbers to explain.