CALCULUS 12 Practice Quiz - Sections 3.1 to 3.3

1) Using the <u>definition of the derivative</u>, find $f'(x)$ of $f(x) = \dfrac{1}{\sqrt{2x-3}}$.

Note: No marks will be given if any other techniques to find derivatives are used.

$$\lim_{h\to 0} \frac{\dfrac{1}{\sqrt{2(x+h)-3}} - \dfrac{1}{\sqrt{2x-3}}}{h}$$

$$\lim_{h\to 0} \frac{\dfrac{\sqrt{2x-3}-\sqrt{2(x+h)-3}}{(\sqrt{2(x+h)-3})(\sqrt{2x-3})}}{h} \times \frac{\sqrt{2x-3}+\sqrt{2(x+h)-3}}{\sqrt{2x-3}+\sqrt{2(x+h)-3}}$$

$$= \lim_{h\to 0} \frac{\cancel{2x}\cancel{-3}\cancel{-2x}-2h\cancel{+3}}{(\sqrt{2(x+h)-3})(\sqrt{2x-3})(\sqrt{2x-3}+\sqrt{2(x+h)-3})} \times \frac{1}{h}$$

$$= \lim_{h\to 0} \frac{-2\cancel{h}}{(\sqrt{2(x+h)-3})(\sqrt{2x-3})(\sqrt{2x-3}+\sqrt{2(x+h)-3})} \times \frac{1}{\cancel{h}}$$

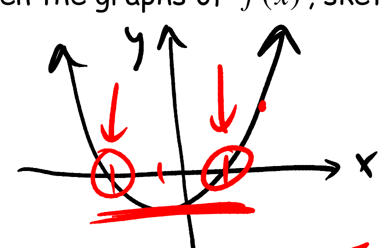
$$= \frac{-\cancel{2}}{(2x-3)(\cancel{2}\sqrt{2x-3})} = \boxed{\frac{-1}{(2x-3)\sqrt{2x-3}}} = \frac{-1}{(2x-3)^{3/2}} = -(2x-3)^{-3/2}$$

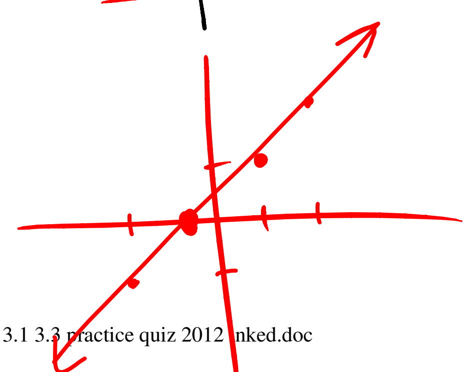
2) What is the derivative of the function in question #1 when x = 2.

$f'(2) = -1$

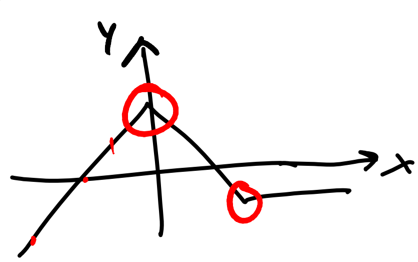
3) Given the graphs of $f(x)$, sketch a graph of $f'(x)$

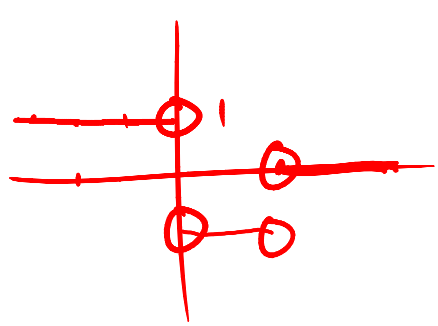
a)





b)





4) Find the derivative of the following polynomials.

a. $f(x) = 6x^6 + \frac{\sqrt{10}}{x^2} - \sqrt{3}x$

$= 6x^6 + \sqrt{10}x^{-2} - \sqrt{3}x^1$

$f'(x) = 36x^5 - 2\sqrt{10}x^{-3} - \sqrt{3}$

b. $f(x) = x^3 - \frac{1}{\sqrt[5]{x^6}} + \pi$

$= x^3 - x^{-6/5} + \pi$

$f'(x) = 3x^2 + \frac{6}{5}x^{-11/5}$

c. $f(x) = (x^3 - x)(\frac{1}{x^2} - \frac{3}{x} + 2)$

(use product rule for question "c" - please simplify)

$f'(x) = (x^3-x)(-2x^{-3} + 3x^{-2})$

$+ (3x^2-1)(x^{-2} - 3x^{-1} + 2)$

$= -2 + \boxed{3x} + 2x^{-2} - 3x^{-1} +$

$3 \boxed{-9x} + 6x^2 - x^{-2} + \cancel{3x^{-1}} - 2 = -1 - 6x + 6x^2 + x^{-2}$

d. $f(x) = \frac{x^2+1}{3x^2 - 2x + 1}$

(please simplify)

$f'(x) = \frac{(3x^2 - 2x + 1)\cdot 2x - (x^2+1)(6x-2)}{(3x^2-2x+1)^2}$

$\frac{6x^3 - 4x^2 + 2x - (6x^3 - 2x^2 + 6x - 2)}{(3x^2-2x+1)^2}$

$= \cancel{6x^3} - 4x^2 + 2x \cancel{-6x^3} + 2x^2 - 6x + 2$

$(-2x^2 - 4x + 2)/$(square of bottom)

Sorry, ran out of room for 4d

5) If $f(3) = -2$ and $f'(3) = 4$, find $g'(3)$ if $g(x) = \frac{2x^2 - 1}{f(x)}$

see website

6) If $f(x) = \frac{3}{1 + x + x^2}$, find the equation of the tangent line at x = -2.

$f'(x) = \frac{\cancel{(1+x+x^2)\cdot 0} - 3(1+2x)}{(1+x+x^2)^2}$

$f'(-2) = \frac{-3(1+2(-2))}{(1+(-2)+(-2)^2)^2} = \frac{9}{9} = 1$

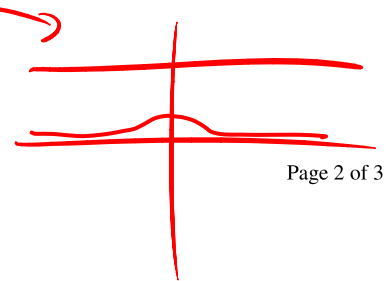
$f(-2) = \frac{3}{1+(-2)+(-2)^2} = \frac{3}{3} = 1$

$y - 1 = 1(x+2)$

7) At which point(s) does the graph of $y = \frac{x}{x^2+4}$ have a slope equal to 2?

* no solution?

$y' = \frac{(x^2+4)\cdot 1 - x(2x)}{(x^2+4)^2} = 2$



8) The cost, in dollars, of producing "x" units of a certain commodity is $C(x) = 920 + 2x - 0.02x^2 + 0.00007x^3$.

i. Find $C'(101)$. ii. Explain what $C'(x)$ means.

i) $C'(x) = 2 - 0.04x + 0.00021x^2$

$C'(101) = 2 - 0.04(101) + 0.00021(101)^2$

$= 0.10221$

ii) $C'(x)$ is the marginal cost, how much it would cost to produce an additional item when you've already produced x items.

9) For the function $f(x) = \begin{cases} x^2 - 3x, \ldots x \le 1 \\ 5 - 2x, \ldots 1 < x < 4 \\ \sqrt{x} - 4x, \ldots x \ge 4 \end{cases}$, where is the function not differentiable? Justify your answer.

- check at $x = 1$ → $f'_-(x) = 2x - 3$ → $f'_-(1) = 2(1) - 3 = -1$

$f'_+(x) = -2$

$f'_-(1) \ne f'_+(1)$, so not differentiable

- check at $x = 4$ $f'_-(x) = -2$

$f'_+(x) = \frac{1}{2}x^{-1/2} - 4$

$= \frac{1}{2}(4)^{-1/2} - 4$

$= 0.25 - 4 = -3.75$

$f'_-(4) \ne f'_+(4)$, so not diff.

∴ not differentiable at $x = 1$, $x = 4$