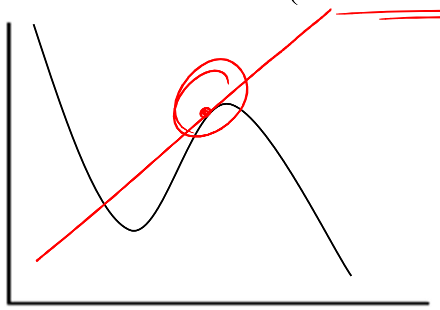
Calculus 12 $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ Name: ___________________________

Ch 2.1 The Tangent and Velocity Problems Date: ___________________________

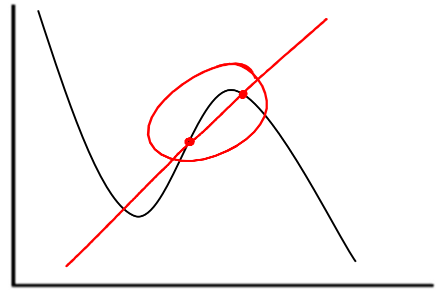
We know how to find the slope of a line, and how to find the slope between two points, but how do you find the slope of a CURVE?

Well, a curve doesn't have just one slope, but you can find the slope at a given part of the curve

Tangent: a line having ONE point of contact with a curve (in the area of interest)

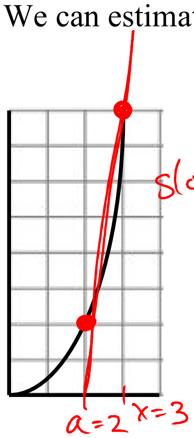


Secant: a line having TWO points of contact with a curve (in the area of interest)

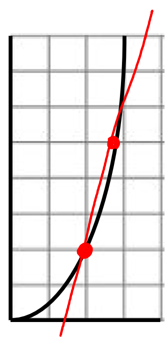


Problem: Find the equation of the tangent line to the parabola $f(x) = x^2$ at the point (1,1).

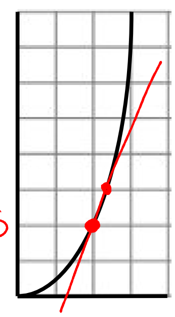
We can estimate the slope with secant lines of points that get closer and closer together



$$\text{slope} = \frac{8-2}{3-2} = \frac{6}{1} = \underline{\underline{6}}$$



$$\text{slope} = \frac{5-2}{2.8-2} = \frac{3}{0.8} = 3.75$$



$$\text{slope} = \frac{3-2}{2.3-2} = \frac{1}{0.3} = 3.\overline{3}$$

As the points come closer together, the slope gets closer and closer to the slope of a tangent line at that point of interest

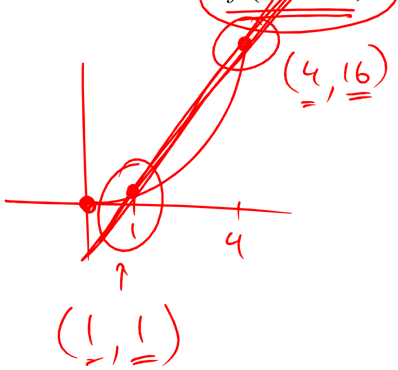
Equation:

$$\text{slope of secant} = \frac{f(x) - f(a)}{x - a} \rightarrow \text{slope of tangent} \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

"the limit as x approaches a"

Problem: Given $f(x) = x^2$, find the average rate of change in y as x changes from 1 to 4.

(secant line slope)

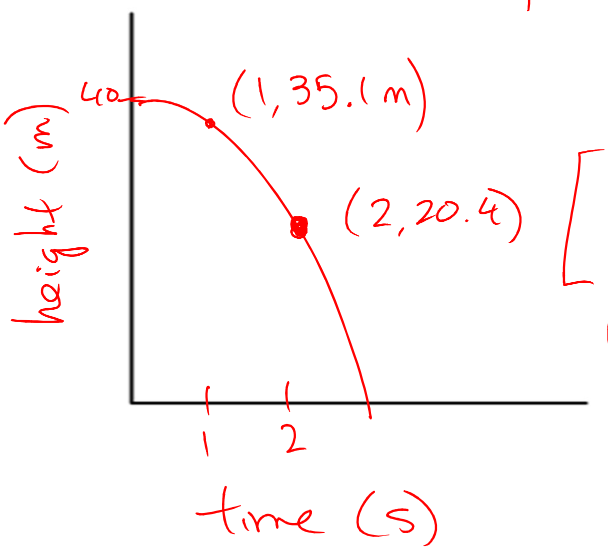


$$\text{slope} = \frac{16-1}{4-1} = \frac{15}{3} = \underline{\underline{5}}$$

The slope of the secant line is like an average rate of change. The slope of the tangent line is like an instantaneous rate of change.

Problem: A ball is dropped from the roof of a building that is 40m tall. If its height above the ground is given by the equation $d = 40 - 4.9\,t^2$, find its velocity at time = 2 seconds.

instantaneous speed, slope of tangent line



x	y
2	20.4
1	35.1
1.9	22.311
1.99	20.59551

$$\text{slope} = \frac{20.4 - 35.1}{2 - 1} = -14.7 \text{ m/s}$$

$$\text{slope} = \frac{22.311 - 20.4}{1.9 - 2} = -19.11 \text{ m/s}$$

$$\text{slope} = \frac{20.4 - 20.59551}{2 - 1.99} = -19.551 \text{ m/s}$$

as time difference is reduced, we get closer & closer to an instantaneous speed of ~ -19.56 m/s

Assignment: $\boxed{\text{Section 2.1 p69 \#2, 4, 6, 8}}$